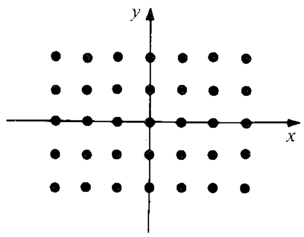
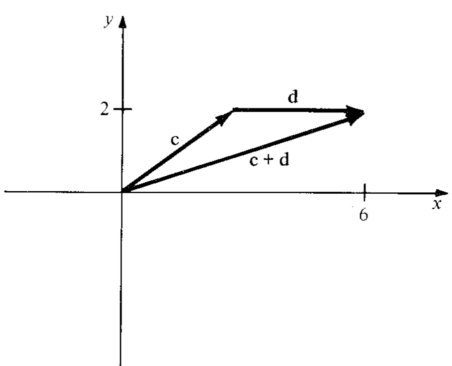
Chapter 13 Answers

13.1 Vectors in the Plane

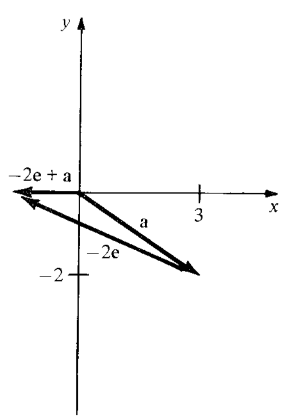
1. $(4, 9)$ **3.** $(-15, 3)$
5. $y = 1$ **7.** No solution
9. No solution **11.** No solution
13. $a = 4$, $b = -1$ **15.** $a = 0$, $b = 1$
17. $(x_1, y_1) + (0, 0) = (x_1 + 0, y_1 + 0) = (x_1, y_1)$
19. $[(x_1, y_1) + (x_2, y_2)] + (x_3, y_3)$
$= (x_1 + x_2 + x_3, y_1 + y_2 + y_3)$
$= (x_1, y_1) + [(x_2, y_2) + (x_3, y_3)]$
21. $a(b(x, y)) = a(bx, by) = (abx, aby) = ab(x, y)$
23.



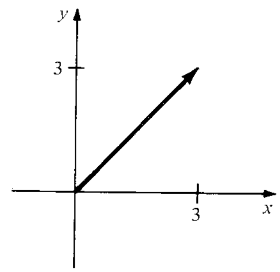
25. (a) $k(1,3) + l(2,0) = m(1,2)$
(b) $k + 2l = m$ and $3k + 0 = 2m$
(c) $k = 4$, $l = 1$ and $m = 6$; i.e. $4SO_3 + S_2 = 6SO_2$
27. (a) **d**
(b) **e**
29. (a) $\mathbf{c} + \mathbf{d} = (6, 2)$



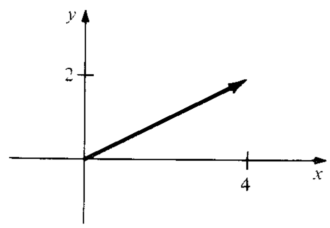
(b) $-2\mathbf{e} + \mathbf{a} = (-1, 0)$



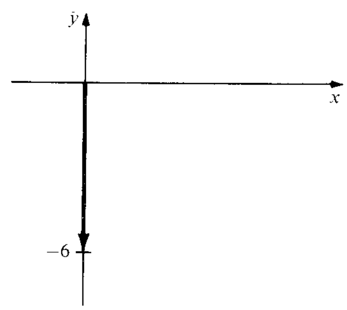
31.



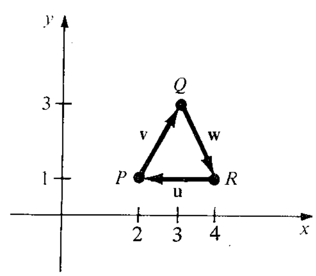
33.



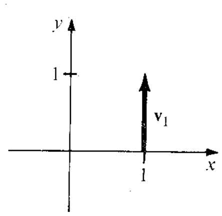
35.



37. (a)

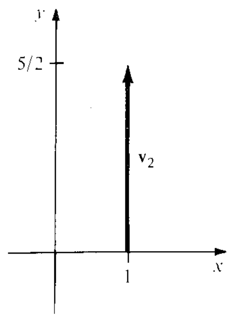


(b) $\mathbf{v} = (1, 2)$; $\mathbf{w} = (1, -2)$; $\mathbf{u} = (-2, 0)$
(c) $\mathbf{0}$
39. (a)

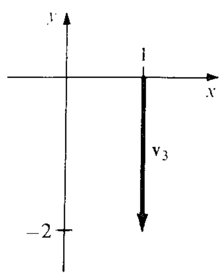


(b) $(0, 1)$

(c) $(0, 5/2)$



(d) $(0, -2)$

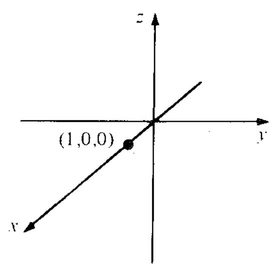


(e) $(1, y)$ (f) $\mathbf{v} = (0, y)$

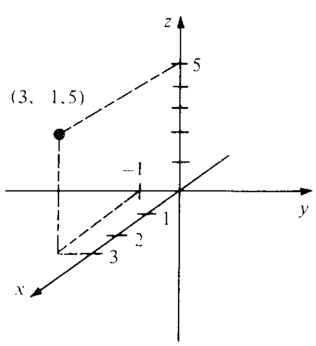
41. (a) Yes (b) $\mathbf{v} = -(s/r)\mathbf{w}$ (c) Eliminate r and s. (d) Solve linear equations.

13.2 Vectors in Space

1.

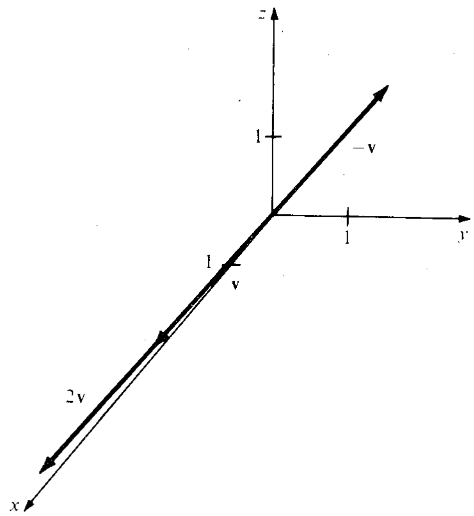


3.

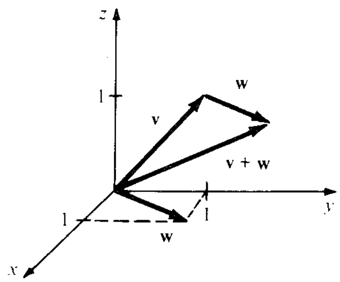


5. $(11, 0, 11)$
7. $(-3, -9, -15)$

9.

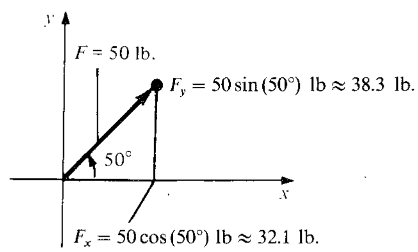


11.



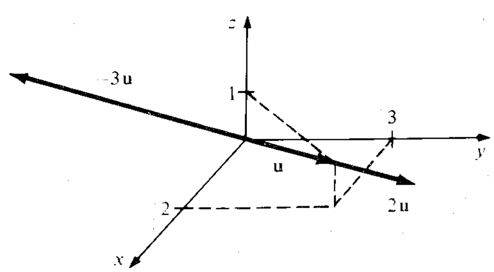
13. $-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ **15.** $7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
17. $\mathbf{i} - \mathbf{k}$ **19.** $\mathbf{i} - \mathbf{j} + \mathbf{k}$
21. $\mathbf{i} + 4\mathbf{j}$, $\theta \approx 0.24$ radians east of north
23. (a) 12:03 P.M.
(b) 4.95 kilometers

25.

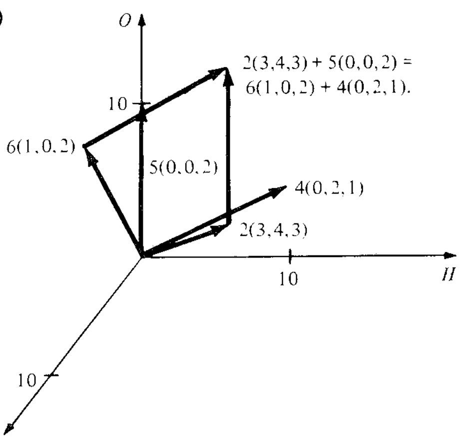


27. The points have the forms $(0, y, 0)$, $(0, 0, z)$, $(x, y, 0)$, and $(x, 0, z)$.

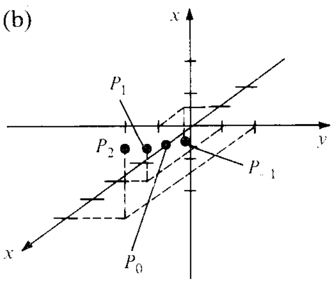
29.



31. $9\mathbf{i} + 12\mathbf{j} + 15\mathbf{k}$
33. $26\mathbf{i} + 16\mathbf{j} + 38\mathbf{k}$
35. $a = \frac{1}{2}, b = -\frac{1}{2}$
37. $a = 5, b = 2$
39. $(4.9, 4.9, 4.9)$ and $(-4.9, -4.9, 4.9)$ newtons.
41. (a) Letting x, y, and z coordinates be the number of atoms of C, H, and O respectively, we get $p(3,4,3) + q(0,0,2) = r(1,0,2) + s(0,2,1)$.
(b) $p = 2, r = 6, s = 4, q = 5$
(c)



43. (a) $P_{-1} = (-1,-1,-1)$, $P_0 = (1,0,0)$, $P_1 = (3,1,1)$, $P_2 = (5,2,2)$
(b)



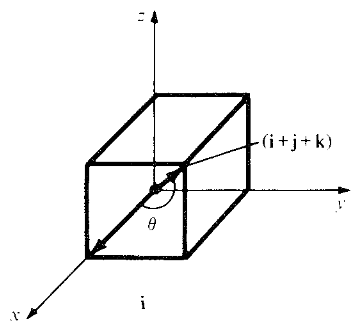
(c) The line through $(1,0,0)$ parallel to the vector $(2,1,1)$.

13.3 Lines and Distances

1. Use vectors with tails at the vertex containing the two sides.
3. Use the distributive law for scalar multiplication.
5. $(1, -\frac{1}{3})$
7. $x = 1 - t, y = 1 - t, z = t$
9. $x = t, y = t, z = t$
11. $x = 1 - t, y = 1 - t, z = t$
13. $x = -1 + 3t, y = -2 - 2t$
15. $(-2, -1, 0)$
17. No
19. $\sqrt{3}$
21. $\sqrt{2}$
23. $2\sqrt{2}$
25. $\pm\sqrt{246}$
27. $\|2\mathbf{i} + \mathbf{j} + 2\mathbf{k}\| = 3$, which is less than $\sqrt{3} + \sqrt{2}$.
29. One solution is $\mathbf{u} = \mathbf{i}$, $\mathbf{v} = -\mathbf{i}$, $\mathbf{w} = \mathbf{i}$.
31. Each side has length $\sqrt{2}$.
33. $(1/\sqrt{3})\mathbf{i} + (1/\sqrt{3})\mathbf{j} + (1/\sqrt{3})\mathbf{k}$, $(1/\sqrt{2})\mathbf{i} + (1/\sqrt{2})\mathbf{k}$
35. $\sqrt{3}$
37. $\sqrt{2}$
39. (i) $\sqrt{14t^2 - 12t + 4}$
(ii) $t = 3/7$
(iii) $\sqrt{10/7}$
41. 13 knots
43. Solve one equation for t and substitute. The line is vertical when $x_1 = x_2$.
45. When the angle between the vectors is 0.

13.4 The Dot Product

1. 4
3. 0
5. ≈ 0.34 radian
7. $\pi/2$ radians
9. $(1/\sqrt{5})\mathbf{i} + (2/\sqrt{5})\mathbf{j}$
11. 0.955 radians



13. Use Figure 13.4.2.
15. $\sqrt{42}$
17. $\sqrt{50/11}$
19. $x + y + z = 0$
21. $x = 0$
23. $-x + y + z - 1 = 0$
25. $(2/\sqrt{14})\mathbf{i} + (3/\sqrt{14})\mathbf{j} + (1/\sqrt{14})\mathbf{k}$
27. $(1/\sqrt{2})\mathbf{i} + (1/\sqrt{2})\mathbf{j}$
29. $x + y + z - 1 = 0$
31. $x = 1 + t, y = 1 + t, z = 1 + t$
33. $(1, -1/2, 3/2)$, $\sqrt{14}/2$
35. $x = (22 - 9t)/7, y = (-6 - 2t)/7, z = t$
37. $3\sqrt{3}$.
39. $\sqrt{2}/2$.
41. Letting (a,b) and (c,d) be the given points, the equation of the line is $(a - c)x + (b - d)y = (1/2)(a^2 - c^2 + b^2 - d^2)$. Use this to show that the two points are equidistant from points on the line.
43. (a) 3
(b) -2
(c) $2\sqrt{3}$
(d) 3
45. Letting $P_1 = (p,q)$ and $P_2 = (r,s)$, we have $(r - p)x + (s - q)y = (r^2 + s^2 - p^2 - q^2)/2$.

47. To show that $\mathbf{v}$ and $\mathbf{w} = (\mathbf{v}\cdot\mathbf{e}_1)\mathbf{e}_1 + (\mathbf{v}\cdot\mathbf{e}_2)\mathbf{e}_2$ are equal, show that $\mathbf{v}-\mathbf{w}$ is orthogonal to both $\mathbf{e}_1$ and $\mathbf{e}_2$.

49. $\mathbf{F}_1 = -(F/2)(\mathbf{i}+\mathbf{j})$ and $\mathbf{F}_2 = (F/2)(\mathbf{i}-\mathbf{j})$

51. (a) $\mathbf{F} = (3\sqrt{2}\,\mathbf{i} + 3\sqrt{2}\,\mathbf{j})$
(b) ≈ 0.322 radians
(c) $18\sqrt{2}$

53. Use the component formula for the dot product.

55. (a) $[12.5)^2 + (16.7)^2 - (20.9)^2]/[(12.5)(16.7)]$ is close to 0.
(b) 0.54%

57. (a) Let $s = t\sqrt{a^2+b^2+c^2}$
(b) Use the fact that $\|\mathbf{u}\| = 1$.
(c) Use $\|\mathbf{u}\|^2 = 1$.
(d) For L_1 and L_2,
$\cos\alpha = 1/\sqrt{3}$ so $\alpha = \cos^{-1}(1/\sqrt{3})$,
$\cos\beta = 1/\sqrt{3}$ so $\beta = \cos^{-1}(1/\sqrt{3})$,
$\cos\gamma = 1/\sqrt{3}$ so $\gamma = \cos^{-1}(1/\sqrt{3})$.
For L_3 and L_4,
$\cos\alpha = 1/\sqrt{83}$ so $\alpha = \cos^{-1}(1/\sqrt{83})$,
$\cos\beta = 1/\sqrt{83}$ so $\beta = \cos^{-1}(1/\sqrt{83})$
and $\cos\gamma = 9/\sqrt{83}$ so $\gamma = \cos^{-1}(9/\sqrt{83})$.
(e) Only the line $t(1, 1, 1)$.

13.5 The Cross Product

1. $\mathbf{j}+\mathbf{k}$
3. $2\mathbf{i}-2\mathbf{j}+4\mathbf{k}$
5. $9\mathbf{i}+18\mathbf{j}$
7. $6\mathbf{i}-2\mathbf{k}$
9. $-\mathbf{i}+\mathbf{k}$
11. $3\sqrt{2}$
13. 2
15. $-(1/\sqrt{2})\mathbf{j} + (1/\sqrt{2})\mathbf{k}$
17. $-(1/\sqrt{2})\mathbf{i} - (1/\sqrt{2})\mathbf{j}$
19. $(\sqrt{2}/6)\mathbf{i} - (\sqrt{2}/6)\mathbf{j} + (2\sqrt{2}/3)\mathbf{k}$
21. $2x+3y+4z=0$
23. $x-3y+2z=0$
25. $3\sqrt{2}/2$
27. The points are collinear, so the area is zero.
29. Substitute component expressions for $\mathbf{v}_1$ and $\mathbf{v}_2$.
31. The angle between the vectors is $\theta - \psi$. Now use property 1 in the box on p. 679.
33. Use the result of Exercise 32.
35. Show that $\mathbf{M}$ satisfies the defining properties of $\mathbf{R}\times\mathbf{F}$.
37. Show that $n_1(\mathbf{N}\times\mathbf{a})$ and $n_2(\mathbf{N}\times\mathbf{b})$ have the same magnitude and direction.
39. (a) Draw a figure showing the two lines and the plane in the hint.
(b) $\sqrt{2}$
41. If $\mathbf{F}$ is the gravitational force, the gyroscope rotates to the left (viewed from above).

13.6 Matrices and Determinants

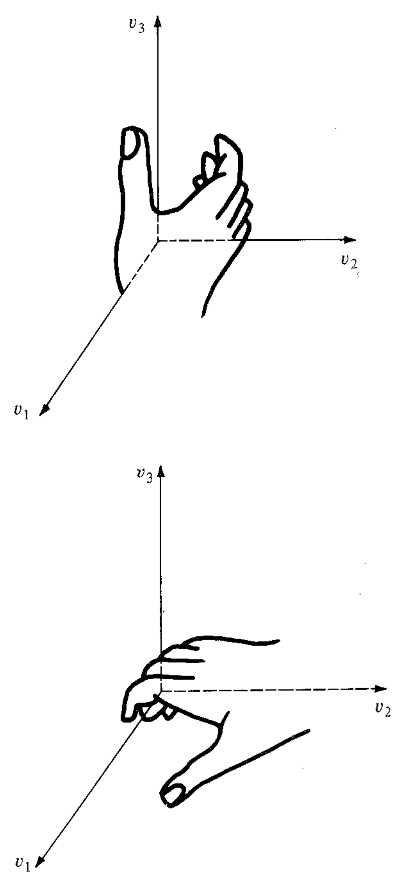
1. 2
3. 0
5. -2
7. 25
9. ac
11. Compute the two determinants.
13. Compute the two determinants.
15. 0
17. 4
19. -6
21. 9
23. abc

25. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = -\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$

27. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

29. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} = -2\mathbf{j}$

31. 6
33. 12
35. Compute and simplify.
37. Compute both determinants and compare.
39. Use Example 8 after renumbering the vectors.
41.



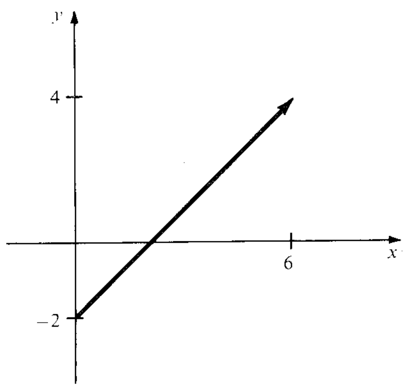
43. Substitute the expressions for x and y in the equations.
45. Substitute the given expressions for x, y, and z in the equations.
47. $x = 37/13$, $y = -3/13$
49. Compute both determinants and compare.

51. Subtract four times row 1 from row 2, subtract seven times row 1 from row 3, expand by column 1 and then evaluate the 2×2 determinant.

53. 3, -6

Review Exercises for Chapter 13

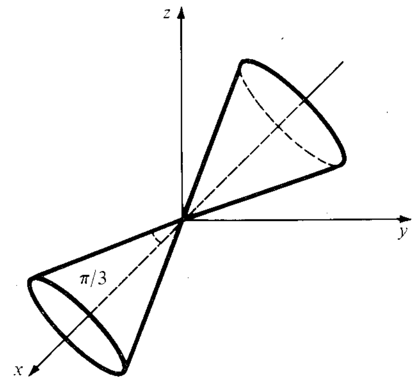
1. $(2, 8)$
3. $(-1, -2, 17)$
5. $11\mathbf{i} + \mathbf{j} - \mathbf{k}$
7. $-4\mathbf{i} + 7\mathbf{j} - 11\mathbf{k}$
9. 6
11. $-2\mathbf{k}$
13. $\mathbf{i} - 2\mathbf{j}$
15. $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$
17. $(-2/\sqrt{22})\mathbf{i} + (3/\sqrt{22})\mathbf{j} + (3/\sqrt{22})\mathbf{k}$
19. 0 (the three vectors lie in a plane).
21. (a) $(6, 6)$



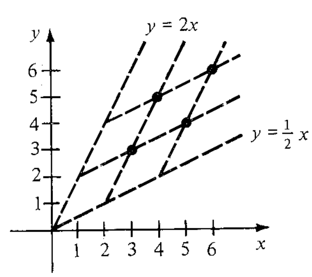
(b) $(9, 7)$

23. Put the triangle in the xy-plane; use cross products with $\mathbf{k}$.
25. $(1825 - 600\sqrt{2})^{1/2} \approx 31.25$ km/hr.
27. (a) $70 \cos\theta + 20 \sin\theta$
(b) $(21\sqrt{3} + 6)$ ft.-lbs.
29. $x = 1 + t, y = 1 + t, z = 2 + t$
31. $x = 1 + t, y = 1 - t, z = 1 - t$
33. $-x + y = 0$
35. $x - y - z - 1 = 0$
37. $x = -t, y = t, z = 3$
39. $x = 2 + t, y = 3 - t, z = 1 - t$
41. $(1/\sqrt{38})\mathbf{i} - (6/\sqrt{38})\mathbf{j} + (1/\sqrt{38})\mathbf{k}$
43. $(2/\sqrt{5})\mathbf{i} - (1/\sqrt{5})\mathbf{j}$
45. $(\sqrt{3}/2)\mathbf{i} + (1/2\sqrt{2})\mathbf{j} + (1/2\sqrt{2})\mathbf{k}$
47. It is parallel to the z-axis.

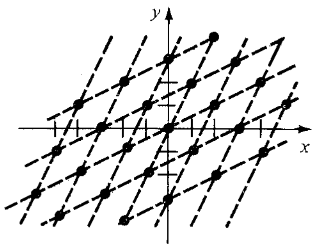
49. This is a (double) cone with vertex at the origin.



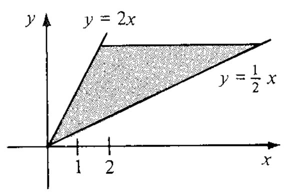
51. (a) Draw a vector diagram. (b) Use $\mathbf{c} \times \mathbf{c} = \mathbf{0}$.
(c) Use part (b).
53. Use the dot product to show that the vectors $\mathbf{a} - \mathbf{b}$ and $-\mathbf{a} - \mathbf{b}$ are perpendicular.
55. 3
57. 1
59. -2
61. 0
63. $\sqrt{381}$
65. $29/2$
67. (a) $\dfrac{1}{6}\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
(b) $1/3$
69. Use the fact that $\|\mathbf{a}\|^2 = \mathbf{a} \cdot \mathbf{a}$, expand both sides and use the definition of $\mathbf{c}$.
71. $x = 3/7, y = -29/21, z = 23/21$
73. -162
75. Each side equals
$2xy - 7yz + 5z^2 - 48x + 54y - 5z - 96$.
(Or switch the first two columns and then subtract the first row from the second.)
77. $\mathbf{v}$ is orthogonal to $\mathbf{i}$
79. (a) $4\mathbf{k}$
(b) $20\sqrt{2}\,\mathbf{i} + 20\sqrt{2}\,\mathbf{j}$
81. (a) Substitute $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ for $\mathbf{w}$.
(b) $(\mathbf{u} - \mathbf{v}) \cdot \mathbf{w} = 0$.
(c) Repeat the reasoning in (a)
(d) Apply (c) to $\mathbf{u} - \mathbf{v}$.
83. (a)



(b)

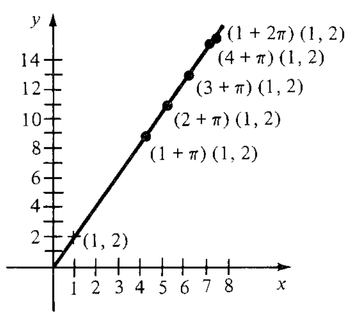


(c)

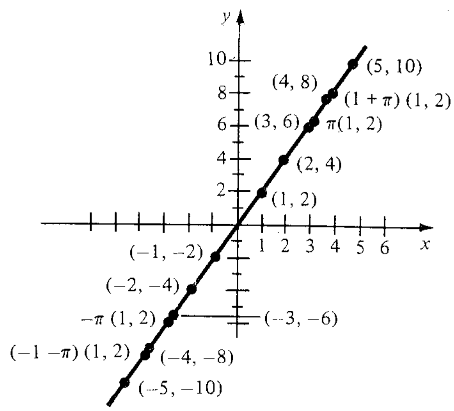


(d) The set is the entire plane.

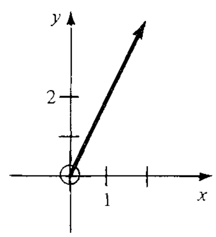
85. (a)



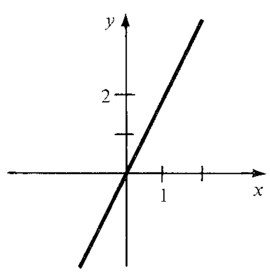
(b)



(c)



(d)

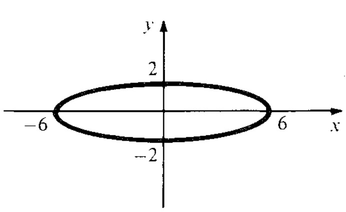


87. $\theta = \sin^{-1}(\sqrt{8}/3)$

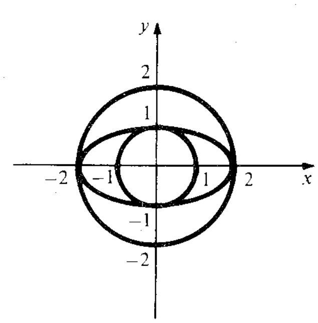
Chapter 14 Answers

14.1 The Conic Sections

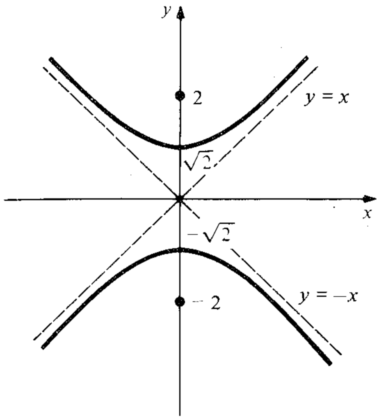
1. Foci at $(\pm 4\sqrt{2}, 0)$.



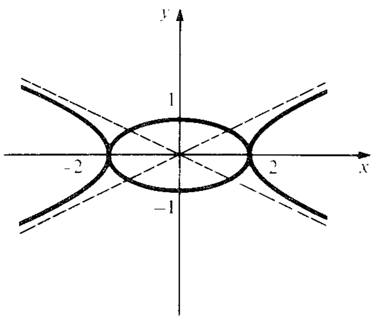
3.



5.



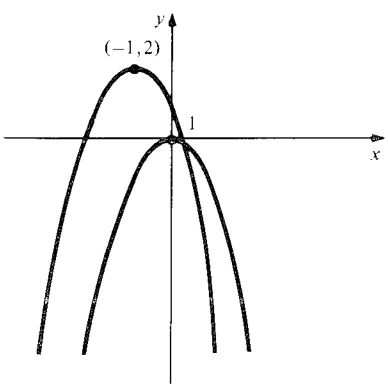
7.



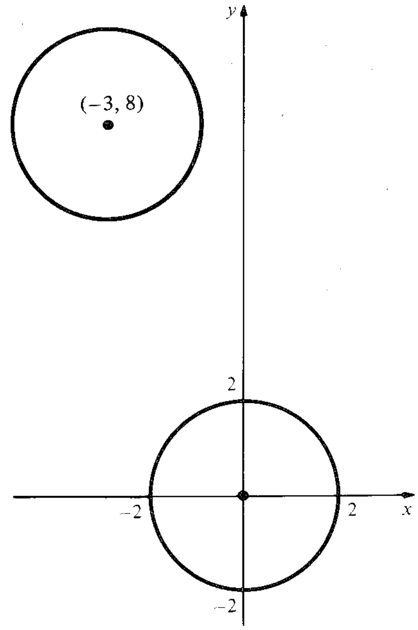
9. $y = x^2/16$
11. $(0, 1/4)$, $y = -1/4$
13. $(1/4, 0)$, $x = -1/4$
15. $x^2 + y^2 = 25$
17. $y = x^2/4$
19. $y^2 - \dfrac{x^2}{3} = 1$
21. On the axis, $2/15$ meters from the mirror.
23. $4/\sqrt{15}$
25. Use the dot product to find an expression for the cosine of the incident and reflected angles.

14.2 Translation and Rotation of Axes

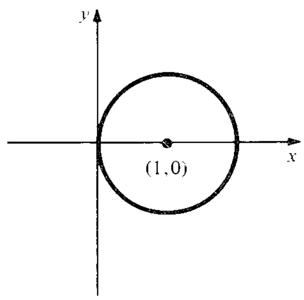
1.



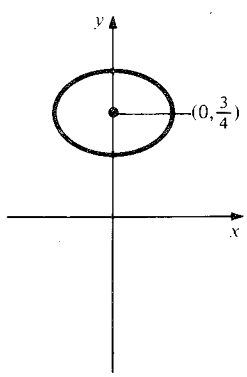
3.



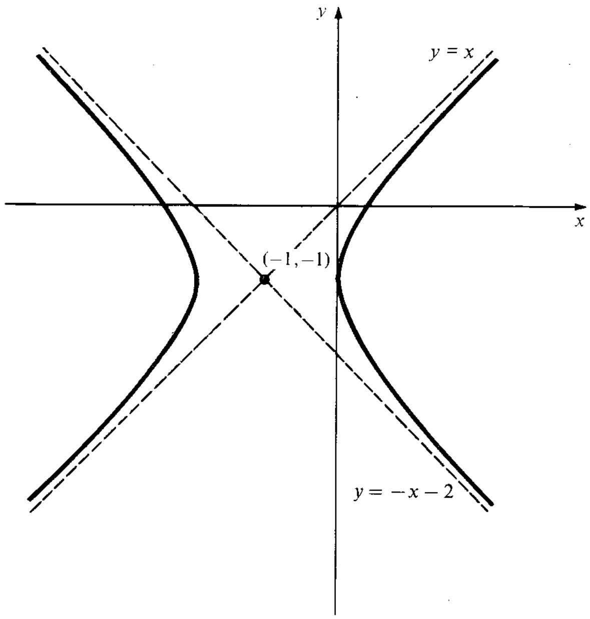
5. A circle of radius 1 centered at $(1, 0)$



7. An ellipse shifted to $(0, 3/4)$.



9. A hyperbola with asymptotes $y = \pm x$, shifted to $(-1, -1)$.



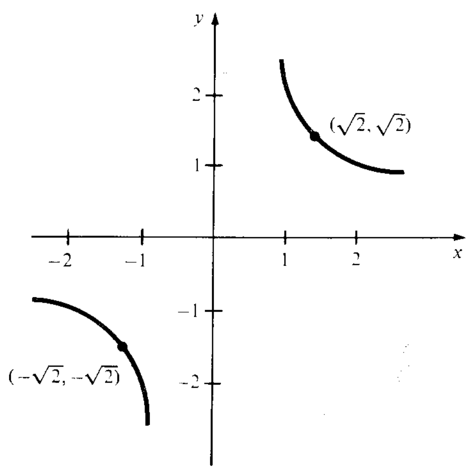
11. $x = X/2 - \sqrt{3}\,Y/2$, $y = \sqrt{3}\,X/2 + Y/2$, $X = x/2 + \sqrt{3}\,y/2$, $Y = -\sqrt{3}\,x/2 + y/2$.

13. $x = 0.97X - 0.26Y$, $y = 0.26X + 0.97Y$, $X = 0.97x + 0.26y$, $Y = -0.26x + 0.97y$.

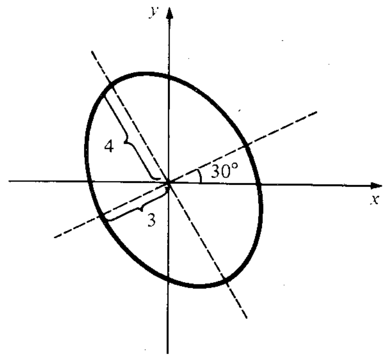
15. Hyperbola

17. Ellipse

19.



21.



23. $x^2 + y^2 - 4x - 6y = 12$

25. $y = (x-1)^2$ **27.** $(y-1)^2 - \dfrac{x^2}{3} = 1.$

29. $X^2 + Y^2 + (1 - \sqrt{3})X + (-1 - \sqrt{3})Y = 2.$

31. For translations, $A = \bar{A}$ and $C = \bar{C}$. For rotations use equations (9) to compute $A + C$.

33. The area of the rotated ellipse is $\pi\sqrt{\dfrac{1}{\bar{A}}} \cdot \sqrt{\dfrac{1}{\bar{C}}}$.

14.3 Functions, Graphs and Level Surfaces

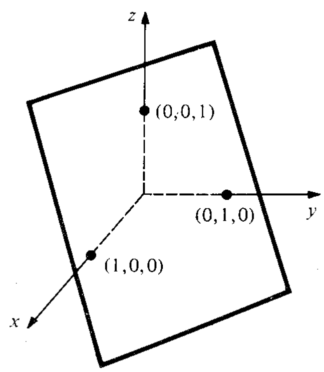
1. All (x, y) with $x \neq 0$; 0, 1.

3. All (x, y) with $x^2 + y^2 \neq 1$; 2, 0.

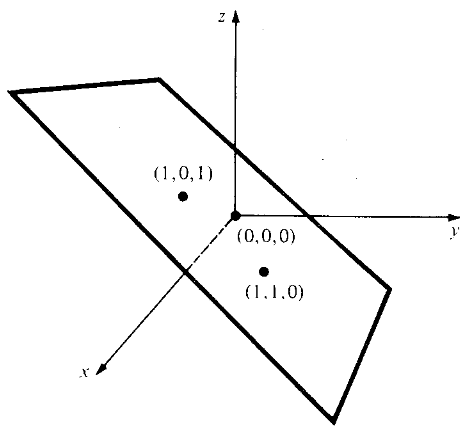
5. All (x, y, z) with $x^2 + y^2 + z^2 \neq 1$; 1, $-2/3$

7. All (x, y) with $x \neq \pi + 2n\pi$, n an integer; $-\sqrt{2}/4$, $\pi(2 - \sqrt{2})/2$.

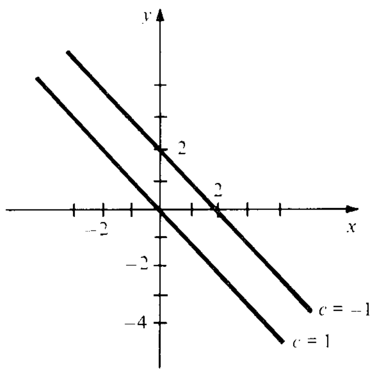
9.



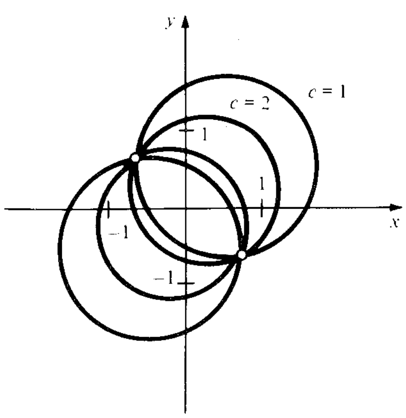
11.



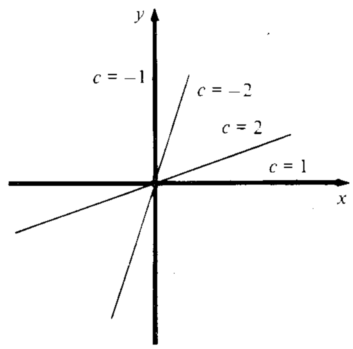
13.



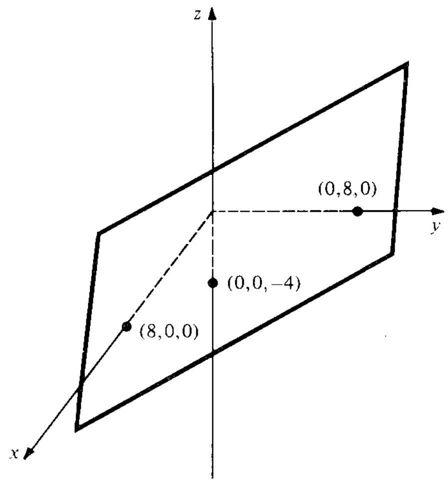
15. Circles with centers on the line $y = x$ and passing through the points $(\pm\sqrt{2}/2, \mp\sqrt{2}/2)$, excluding points on the circle $x^2 + y^2 = 1$.



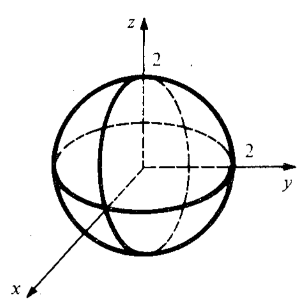
17. Lines through the origin, excluding points on the line $x = y$.



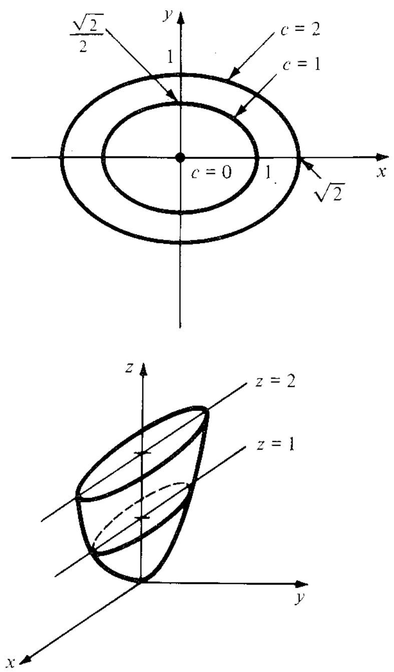
19.



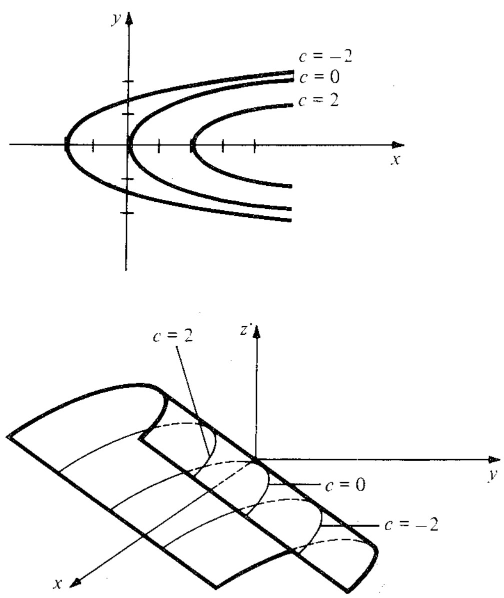
21.



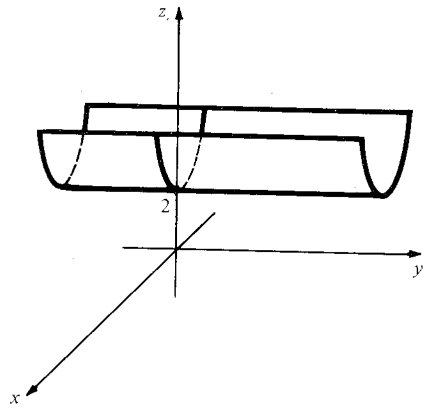
23.



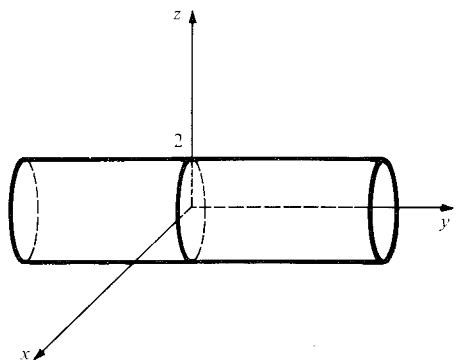
25.



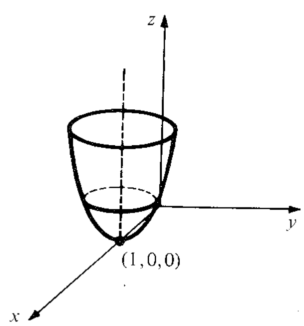
27.



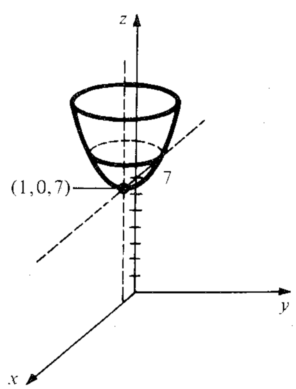
29.



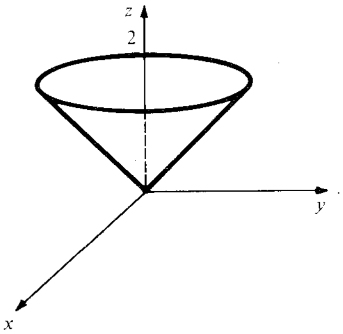
31.



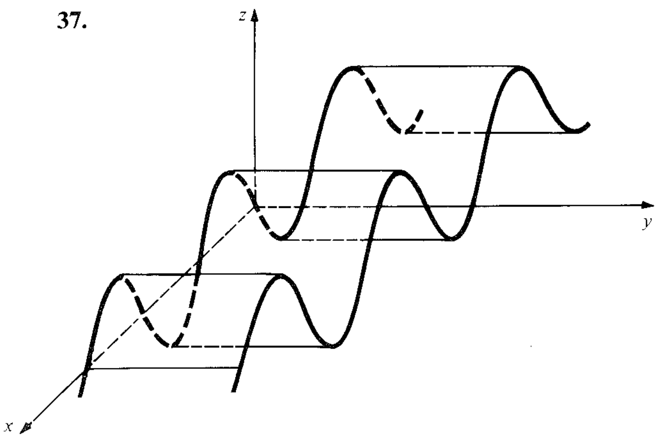
33.



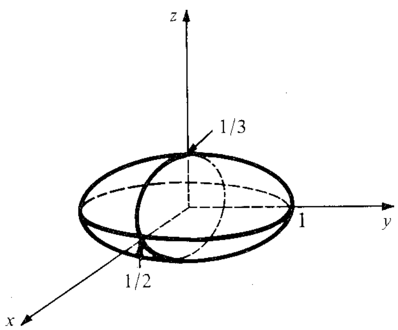
35.



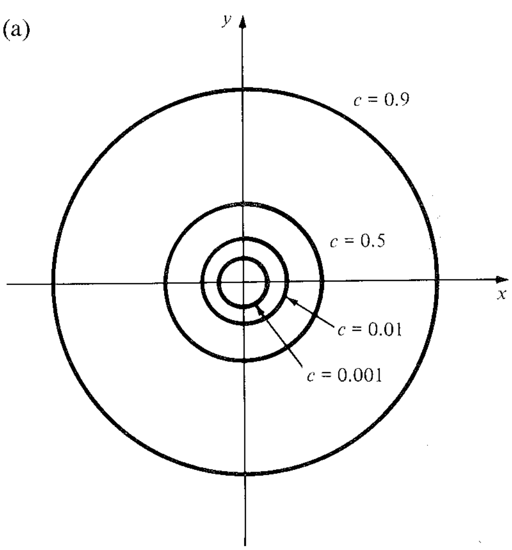
37.



39.

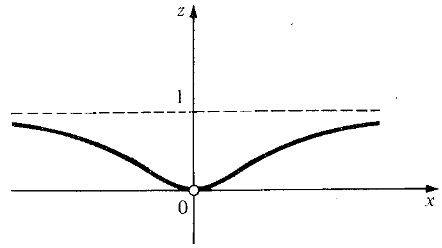


41. (a)



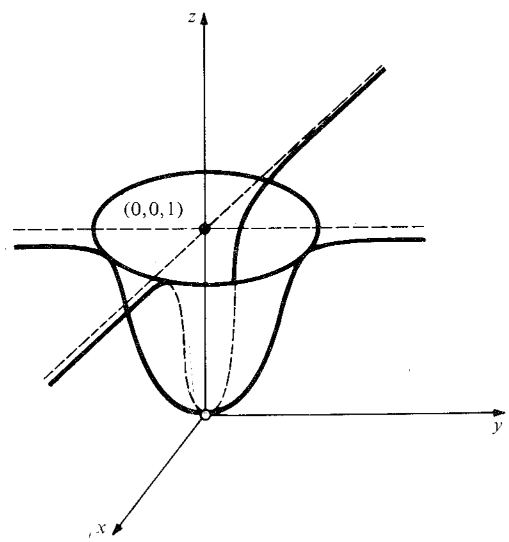
(b) No level curve.

(c)



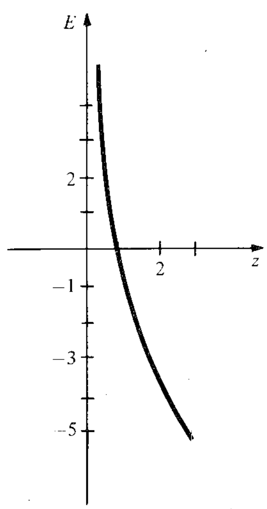
(d) In polar coordinates, the equation is $f(r, \theta) = e^{-1/r^2}$. This is independent of θ.

(e) The graph looks like a plane gradually sloping down to a pit in the center.



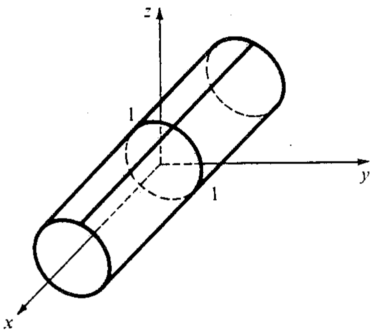
43. (a) $z = \exp[(-12)(x+y)/25(x-y)]$

(b)

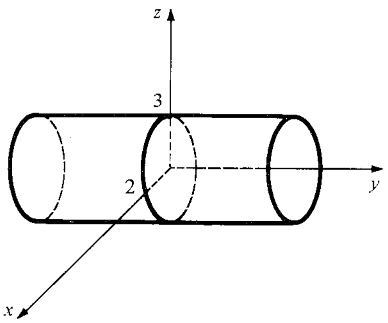


14.4 Quadric Surfaces

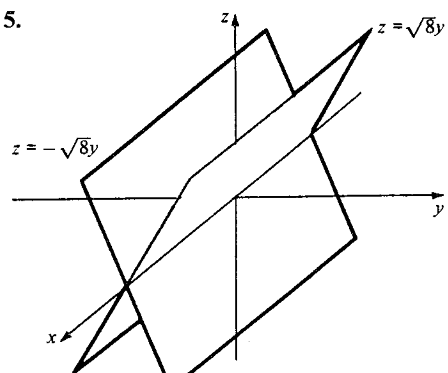
1.



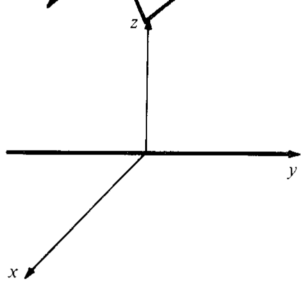
3.



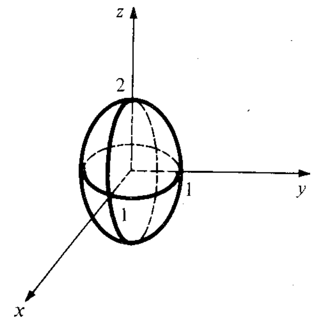
5.



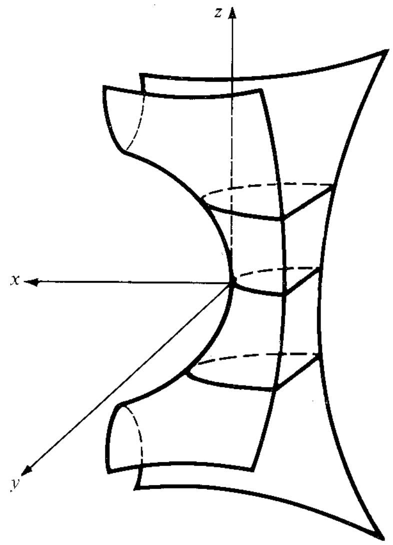
7.



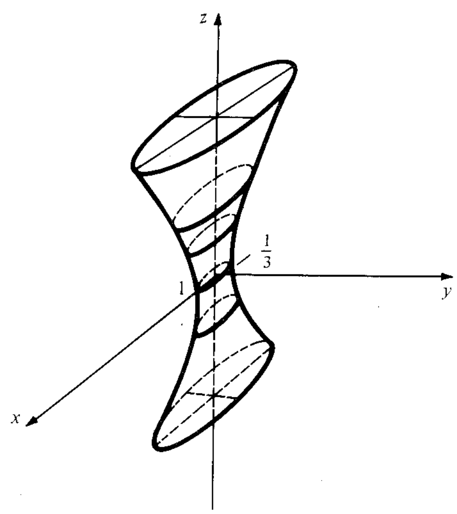
9.



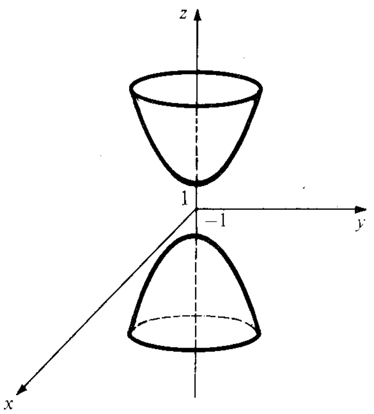
11.



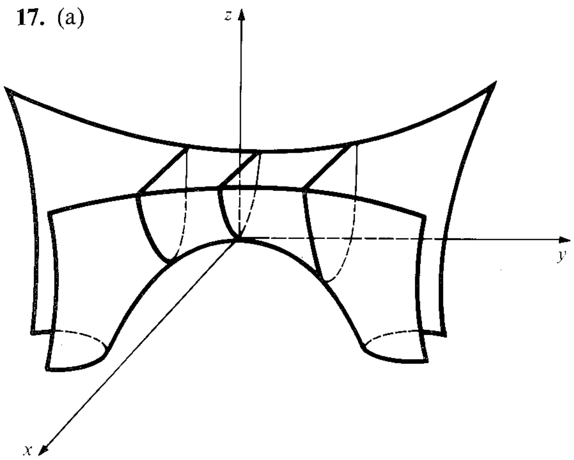
13.



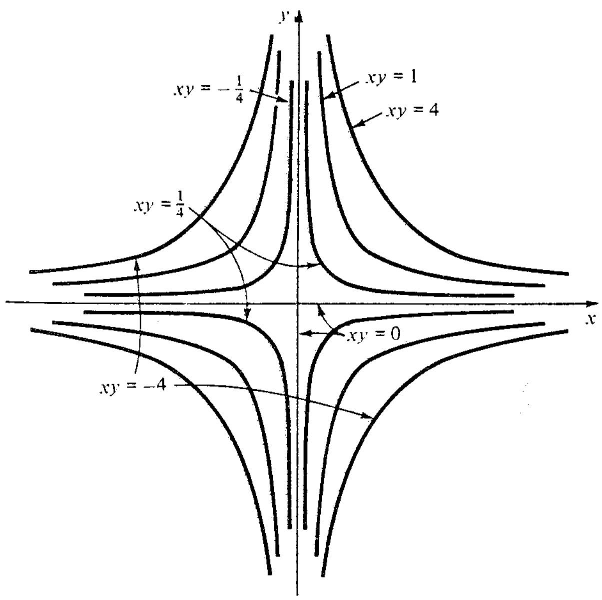
15.



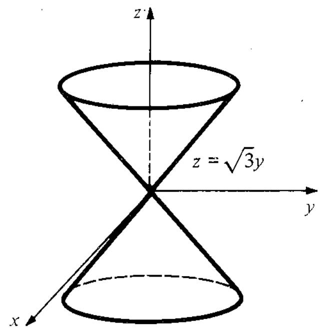
17. (a)



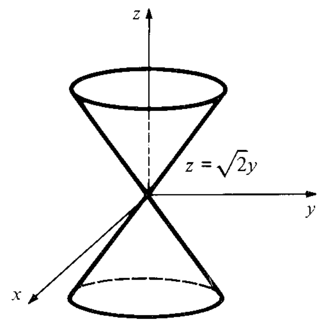
(b) Rotate the x and y axes by 45°.



19.



21.



23. Substituting $z = 1$ gives $x^2 + y^2 = 1$, a circle.

25. (a) They are ellipses.

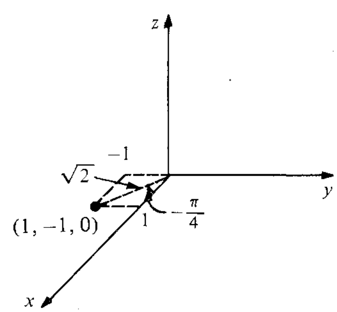
(b) In each case the cross section is two straight lines.

(c) If (x_0, y_0, z_0) satisfies the equation, so does (tx_0, ty_0, tz_0).

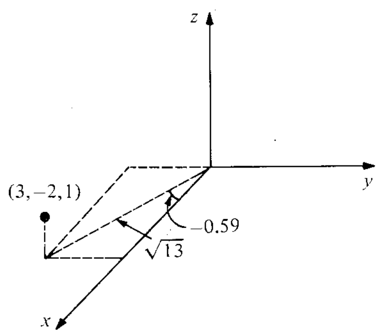
27. Substitute $x = x_0 + \xi u_1 + \eta v_1$, $y = y_0 + \xi u_2 + \eta v_2$, $z = z_0 + \xi u_3 + \eta v_3$ into $x^2 + y^2 = z^2$, where $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$.

14.5 Cylindrical and Spherical Coordinates

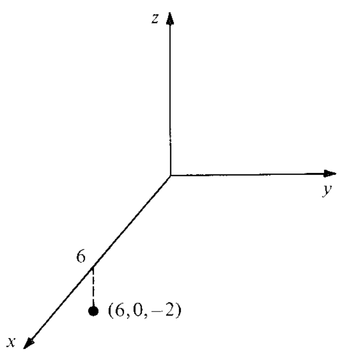
1. $(\sqrt{2}, -\pi/4, 0)$



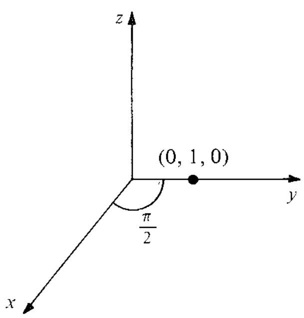
3. $(\sqrt{13}, -0.588, 1)$



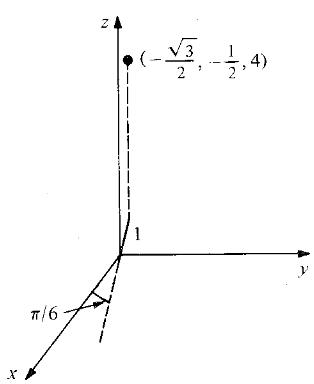
5. $(6, 0, -2)$



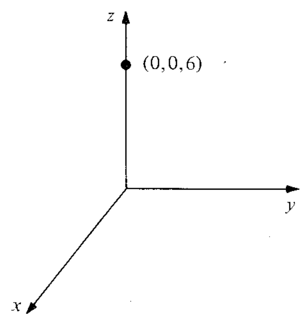
7. $(0, 1, 0)$



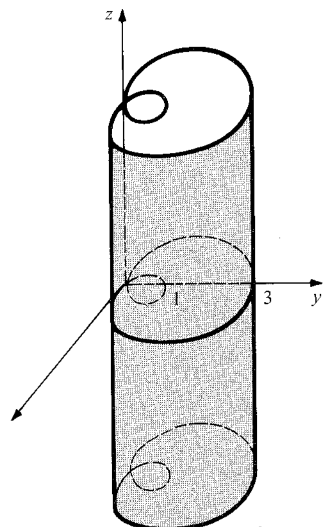
9. $(-\sqrt{3}/2, -1/2, 4)$



11. $(0, 0, 6)$



13.

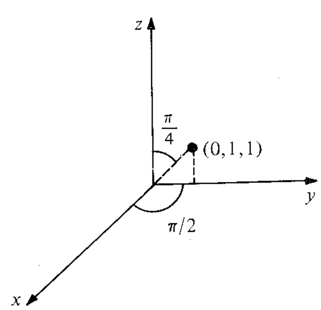


15. Reflection through xy-plane.

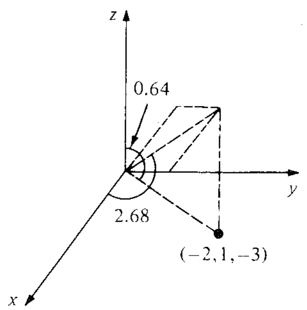
17. Stretching by a factor of 2 away from the z-axis, and a reflection through the xy-plane.

19. Right circular cylinder with radius r; vertical plane making an angle θ with the xz-plane; horizontal plane containing $(0, 0, z)$.

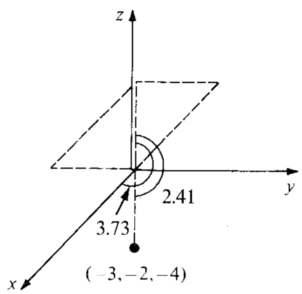
21. $(\sqrt{2}, \pi/2, \pi/4)$



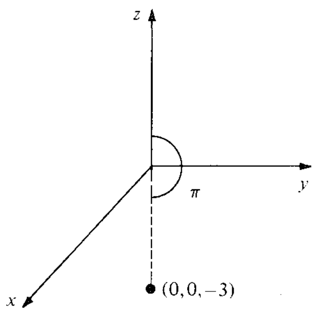
23. $(\sqrt{14}, 2.68, 2.50)$



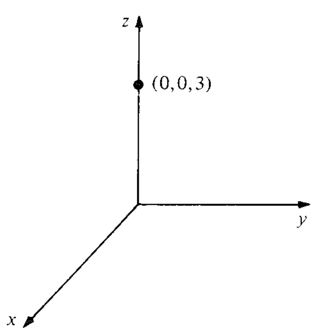
25. $(\sqrt{29}, 3.73, 2.41)$



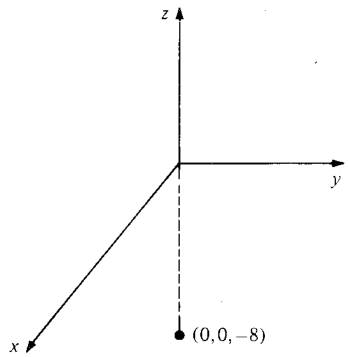
27. $(0, 0, -3)$



29. $(0, 0, 3)$



31. $(0, 0, -8)$



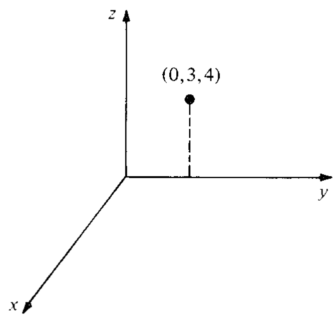
33. $\rho^2 = 2/\sin 2\phi \cos\theta$

35. The vertical half plane with positive y-coordinates and making a 45° angle with the xz-plane.

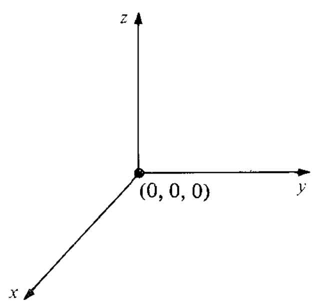
37. It moves each point twice as far from the origin along the same line through the origin.

39. The unit circle in the xy-plane.

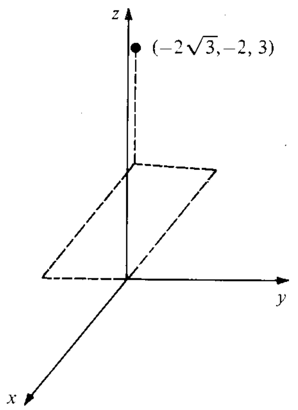
41. $(3, \pi/2, 4)$, $(5, \pi/2, 0.64)$



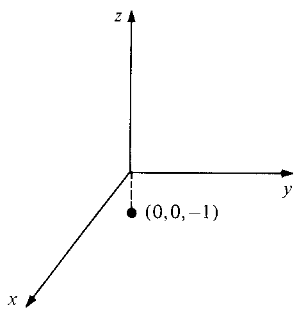
43. $(0, \theta, 0)$, $(0, \theta, \phi)$ for any θ, ϕ.



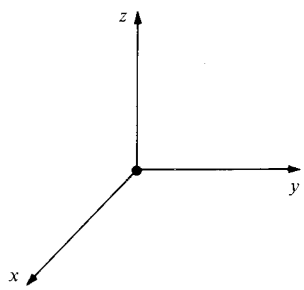
45. $(4, 7\pi/6, 3)$, $(5, 7\pi/6, 0.93)$



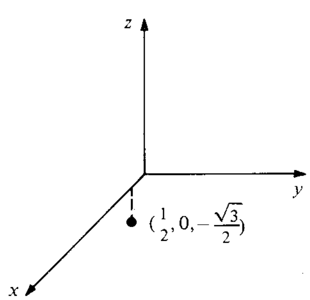
47. $(\sqrt{2}/2, \sqrt{2}/2, 1)$, $(\sqrt{2}, \pi/4, \pi/4)$
49. $(0, 0, 1)$, $(1, \pi/4, 0)$
51. $(0, 2, 1)$, $(\sqrt{5}, \pi/2, 1.11)$
53. $(0, 0, -1)$, $(0, \theta, -1)$ for any θ



55. $(0, 0, 0)$, $(0, \theta, 0)$ for any θ



57. $(1/2, 0, -\sqrt{3}/2)$, $(1/2, 0, -\sqrt{3}/2)$



59. (a) $z = r^2\cos 2\theta$
(b) $1 = \rho \tan\phi \sin\phi \cos 2\theta$

61. (a) The length of $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is $(x^2 + y^2 + z^2)^{1/2} = \rho$
(b) $\cos\phi = z/(x^2 + y^2 + z^2)^{1/2}$
(c) $\cos\theta = x/(x^2 + y^2)^{1/2}$

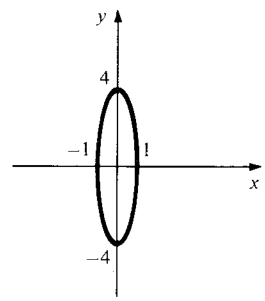
63. $0 \leqslant r \leqslant a$, $0 \leqslant \theta \leqslant 2\pi$ means that (r, θ, z) is inside the cylinder with radius a centered on the z-axis, and $|z| \leqslant b$ means that it is no more than a distance b from the xy plane.

65. $-(d/6)\cos\phi \leqslant \rho \leqslant d/2$, $0 \leqslant \theta \leqslant 2\pi$, and $\pi - \cos^{-1}(1/3) \leqslant \phi \leqslant \pi$.

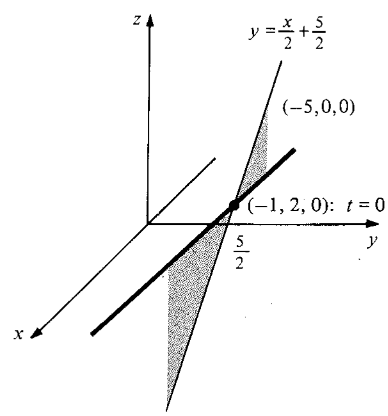
67. This is a surface whose cross-section with each surface $z = c$ is a four-petaled rose. The petals shrink to zero as $|c|$ changes from 0 to 1.

14.6 Curves in Space

1.



3.



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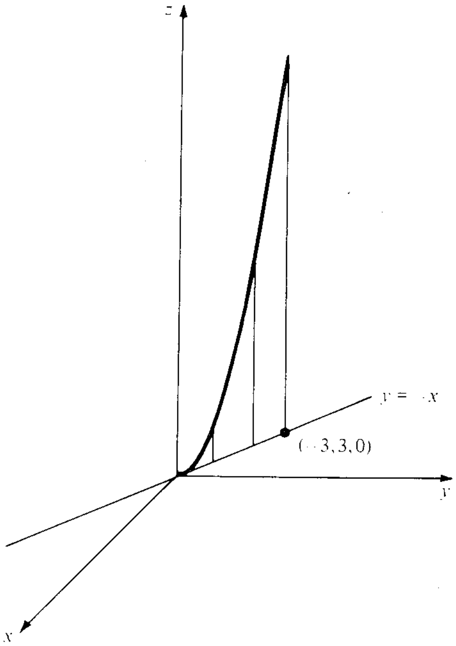
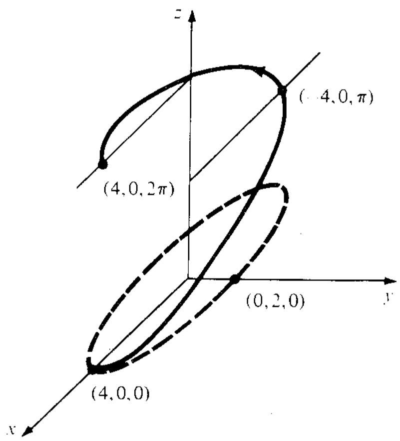
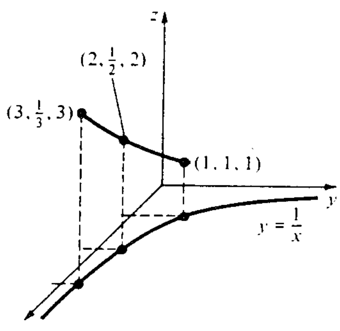


7.



9.



11. (a) An ellipse in the plane spanned by **v** and **w** and passing through the tip of **u**. The ellipse has semi-major axis 4 and semi-minor axis 2.
(b) 1, $-1 + 4\cos t + 8\sin t$, and $4\cos t - 8\sin t$.

13. $\boldsymbol{\sigma}'(t) = -3\sin t\mathbf{i} - 8\cos t\mathbf{j} + e^t\mathbf{k}$;
$\boldsymbol{\sigma}''(t) = -3\cos t\mathbf{i} + 8\sin t\mathbf{j} + e^t\mathbf{k}$.

15. $(e^t - e^{-t})\mathbf{i} + (\cos t - \csc t\cot t)\mathbf{j} - 3t^2\mathbf{k}$.

17. $[t^3\cos t(-2 + \csc^2 t) - 3t^2\sin t(2 + \csc^2 t)]\mathbf{i} +$
$[t^2e^{-t}(3 - t) + 2t^2e^t(t + 3)]\mathbf{j} +$
$[e^t\csc t(1 - \cot t) - e^{-t}(\cos t + \sin t)]\mathbf{k}$

19. $e^t[2e^t\mathbf{i} + (\sin t + \cos t)\mathbf{j} + t^2(3 + t)\mathbf{k}]$

21. $\dfrac{d}{dt}\|\boldsymbol{\sigma}'(t)\|^2 = 2\boldsymbol{\sigma}'(t)\cdot\boldsymbol{\sigma}''(t) = 0.$

23. (a) $\cos t\mathbf{i} - 4\sin t\mathbf{j}$
(b) $-\sin t\mathbf{i} - 4\cos t\mathbf{j}$
(c) $\sqrt{\cos^2 t + 16\sin^2 t}$

25. (a) $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ (b) 0 (c) $\sqrt{6}$

27. (a) $-\mathbf{i} + \mathbf{j} + 2t\mathbf{k}$ (b) $2\mathbf{k}$ (c) $\sqrt{2 + 4t^2}$

29. (a) $-4\sin t\mathbf{i} + 2\cos t\mathbf{j} + \mathbf{k}$ (b) $-4\cos t\mathbf{i} - 2\sin t\mathbf{j}$
(c) $\sqrt{5 + 12\sin^2 t}$

31. (a) $\mathbf{i} - (1/t^2)\mathbf{j} + \mathbf{k}$ (b) $(2/t^3)\mathbf{j}$ (c) $\sqrt{2t^4 + 1}\,/t^2$

33. $(6, 6t, 3t^2)$; $(0, 6, 6t)$; $(x, y, z) = t(6, 0, 0)$

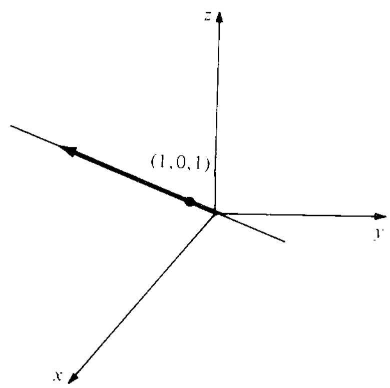
35. $(-2\sin t\cos t, 3 - 3t^2, 1)$; $(-2\cos 2t, -6t, 0)$;
$(x, y, z) = (1, 0, 0) + t(0, 3, 1)$

37. $(\sqrt{2}, e^t, -e^{-t})$; $(0, e^t, e^{-t})$;
$(x, y, z) = (0, 1, 1) + t(\sqrt{2}, 1, -1)$

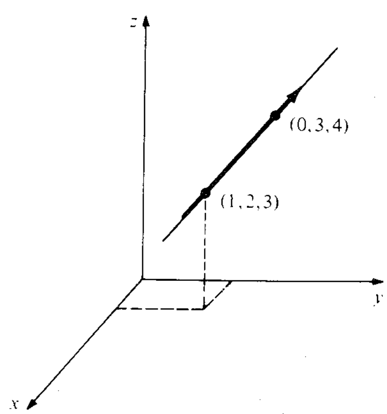
39. $(2e, 0, \cos 1 - \sin 1)$

41. (a) $t(1,0,1)$

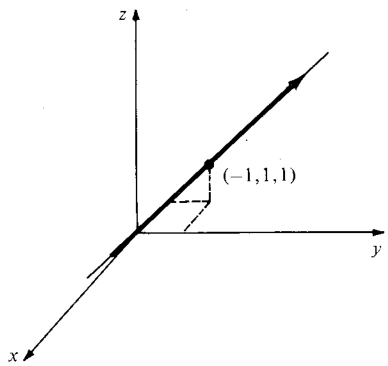
(a) (b)



(b) $(1, 2, 3) + t(-1, 1, 1)$

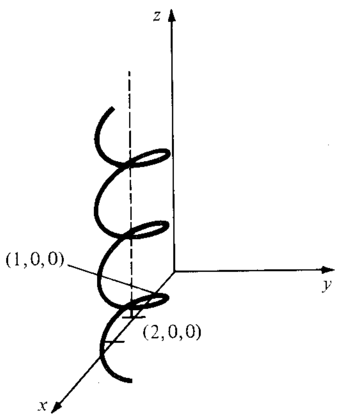


(c) $t(-1, 1, 1)$



43. (a) The curve is a right circular helix with axis parallel to the z-axis.

(b)

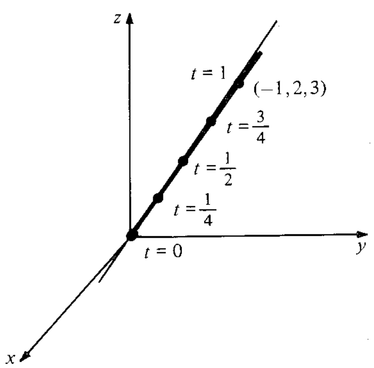


(c) The curve becomes a circle in the xy plane with center $(2, 0, 0)$ and radius 1.

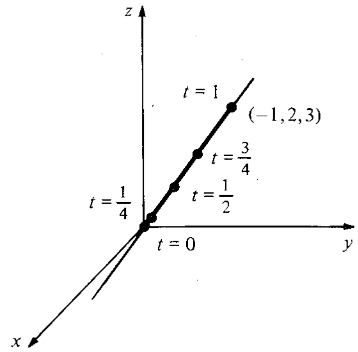
45. (a) Substitute

(b) $(A_1\cos t + B_1\sin t, A_2\cos t + B_2\sin t, A_3\cos t + B_3\sin t)$ where A_1, A_2, A_3, B_1, B_2 and B_3 are constants.

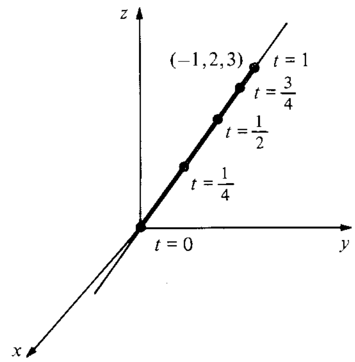
47. (a) (i)



(ii)



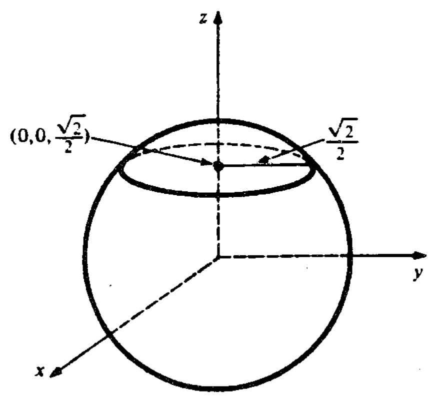
(iii)



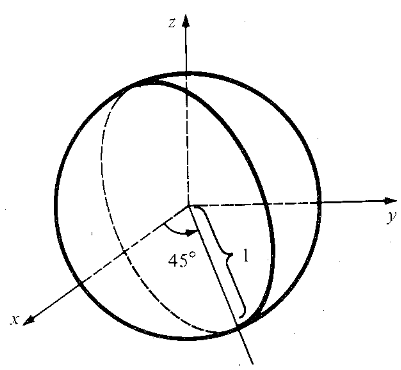
(b) Each curve is the line segment joining $(0, 0, 0)$ to $(-1, 2, 3)$. It is covered once by (i) and (iii) and twice by (ii). The velocity is constant in (i), variable in (ii) and (iii).

49. In each case, verify that $x^2 + y^2 + z^2 = 1$, so the curve lies on the sphere.

(a)



(b)



51. The set of points above or below P_0 have coordinates (x_0, y_0, z) where $z = \cos^{-1}x_0 + 2n\pi$ if $x_0 \geqslant 0$ or $z = -\cos^{-1}x_0 + 2n\pi$ if $x_0 \leqslant 0$, n an integer. The vertical distance is 2π.

53. Let $\boldsymbol{\sigma}_1 = f_1\mathbf{i} + g_1\mathbf{j} + h_1\mathbf{k}$, $\boldsymbol{\sigma}_2 = f_2\mathbf{i} + g_2\mathbf{j} + h_2\mathbf{k}$. Form $\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2$ and differentiate using the sum rule for scalar functions.

55. Using notation in the answer to Exercise 53, form $\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2$ and differentiate using the product rule for scalar functions.

57. (a) $\boldsymbol{\sigma}(t)$ describes a curve in the plane through the origin perpendicular to $\mathbf{u}$.
(b) Same as (a), except that the plane need not go through the origin.
(c) $\boldsymbol{\sigma}(t)$ describes a curve lying in the cone with $\mathbf{u}$ as its axis and vertex angle $2\cos^{-1}b$.

14.7 The Geometry and Physics of Space Curves

1. $2\pi\sqrt{5}$

3. $4\sqrt{2} - 2$

5. ≈ 3.326

7. $-0.32\pi^2\mathbf{r}$, where $\mathbf{r}$ is the vector from the center to the particle.

9. (6.05×10^3) seconds.

11. (a) From $ma = GmM/R^2$, $g = GM/R^2 = (6.67 \times 10^{-11})(5.98 \times 10^{24})/(6.37 \times 10^6)^2 = 9.83\ \mathrm{m/sec}^2$.
(b) The acceleration is $-9.8\mathbf{k}$ if $\mathbf{k}$ points upward.

13. (a) $x'' = (qb/cm)y'$; $y'' = (-qb/cm)x'$; $z'' = 0$.
(b) $x = -(amc/qb)\cos(qbt/mc) + (amc/qb) + 1$, $y = (amc/qb)\sin(qbt/mc)$, $z = ct$.
(c) $r = amc/qb$, the axis is the line parallel to the z-axis through $(amc/qb + 1, 0, 0)$.

15. The circle parametrized by arc length is $\boldsymbol{\sigma}(s) = (r\cos(s/r), r\sin(s/r))$. Calculate $\mathbf{T} = d\boldsymbol{\sigma}/ds$ and $d\mathbf{T}/ds$.

17. $k = 1/\sqrt{2}\,(2y^2 + x^2/2)^{3/2}$.

19. Assume that the curve is parametrized by arc length and show that $\mathbf{v}$ is constant.

21. Force magnitude = (mass) $\times$ (speed)^2 $\times$ (curvature).

23. (a) $\mathbf{n}$ is the normal to the plane. Since $\boldsymbol{\sigma}'$, $\boldsymbol{\sigma}''$, $\boldsymbol{\sigma}'''$ are perpendicular to $\mathbf{n}$, their triple product is zero. By Exercise 22(e), $\tau = 0$.
(b) By Exercise 22(e), $d\mathbf{B}/dt = \mathbf{0}$. By Exercise 22(a), $\mathbf{B}$ lies in the direction of $\mathbf{v} \times \mathbf{a}$.

25. Use the hint for the second equation and Exercise 22 (a) for the third.

14.S Rotations and the Sunshine Formula

1. (a) $\mathbf{m}_0 = (1/\sqrt{6})(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$,
$\mathbf{n}_0 = (1/2\sqrt{3})(\mathbf{i} + \mathbf{j} - \mathbf{k})$
(b) $\mathbf{r} = [\cos(\pi t/12)/2\sqrt{2} + \sin(\pi t/12)/4]\mathbf{i} + [-1/2\sqrt{2} + \cos(\pi t/12)/2\sqrt{2} + \sin(\pi t/12)/4]\mathbf{j} + [-1/2\sqrt{2} + \cos(\pi t/12)/\sqrt{2} - \sin(\pi t/12)/4]\mathbf{k}$
(c) $(x, y, z) = (-1/2\sqrt{2})(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + (-\pi/48)(\mathbf{i} + \mathbf{j} + \mathbf{k})(t - 12)$

3. T_d would be longer.

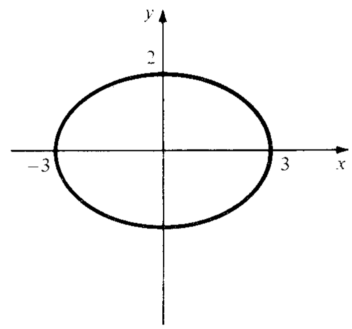
5. The "exact" formula is $-\tan l \sin\alpha = \cos(2\pi t/T_d)[\tan(2\pi t/T_y)\tan(2\pi t/T_d) - \cos\alpha]$.

7. $A = 9.4°$

9. The equator would receive approximately six times as much solar energy as Paris.

Review Exercises for Chapter 14

1.



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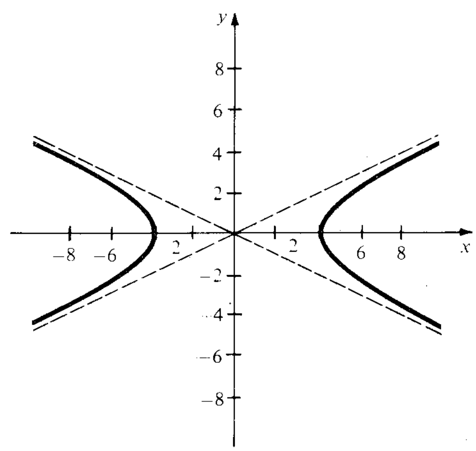

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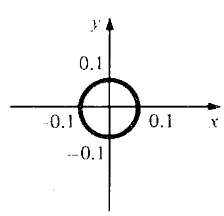

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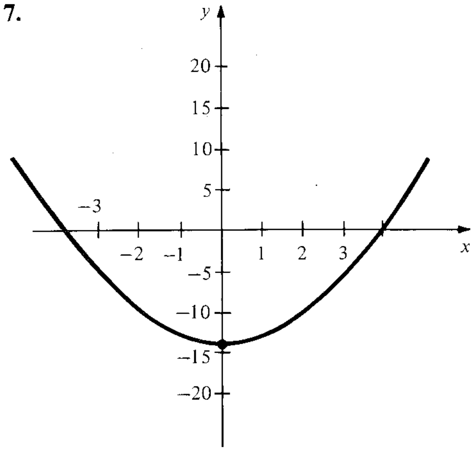

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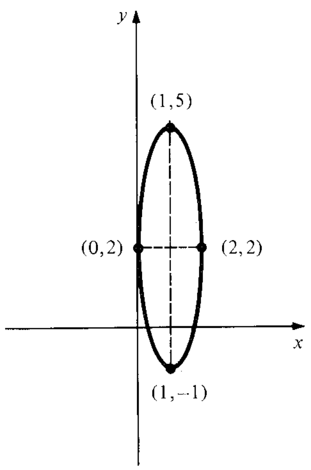

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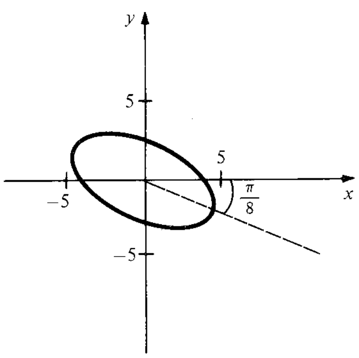

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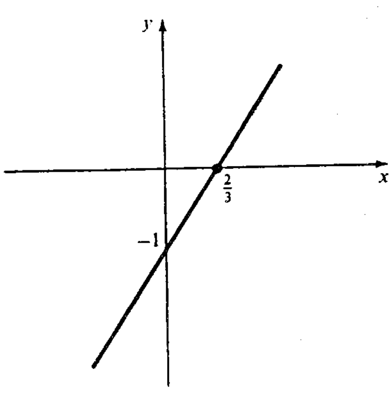

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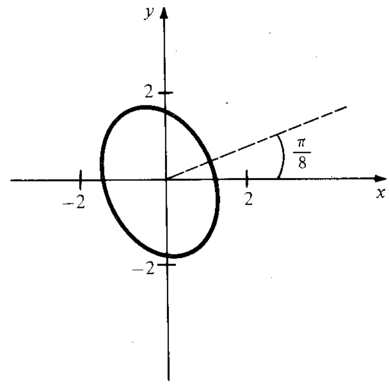
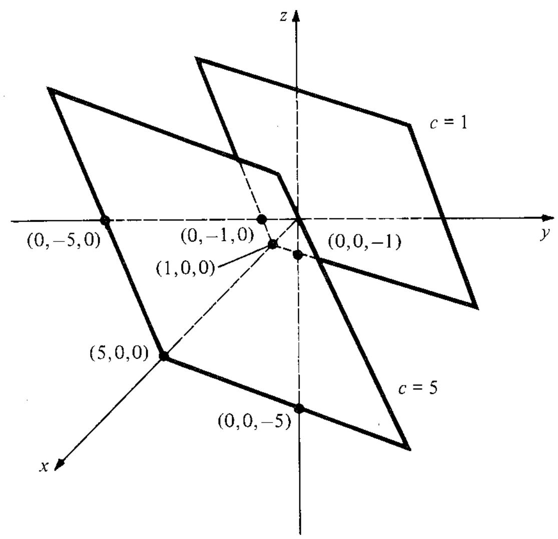
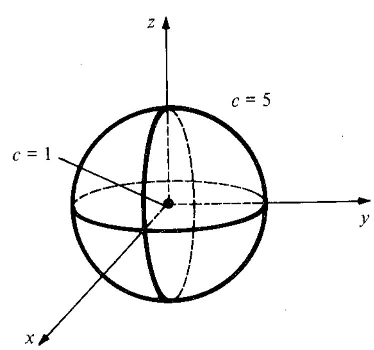


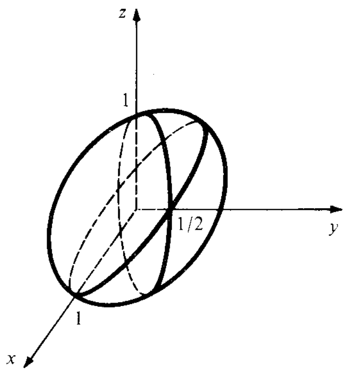
17. The level surfaces are parallel planes.



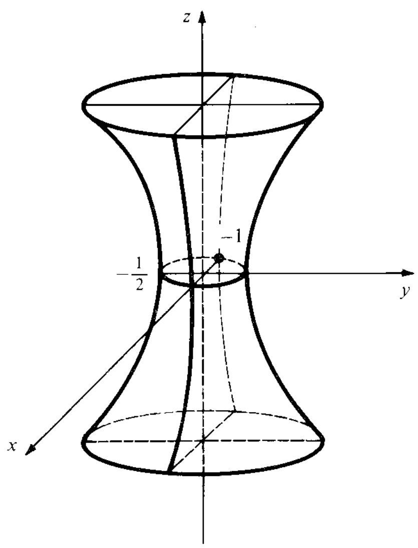
19. The level surfaces are spheres of radius $\sqrt{c-1}$.



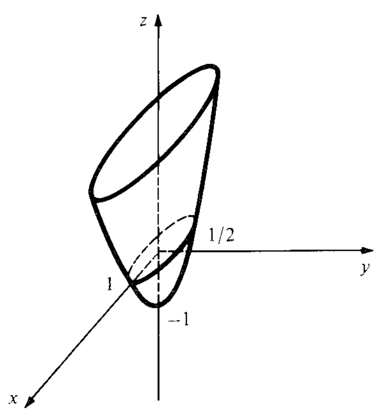
21. Ellipsoid with intercepts $(\pm 1, 0, 0)$, $(0, \pm 1/2, 0)$ and $(0, 0, \pm 1)$.



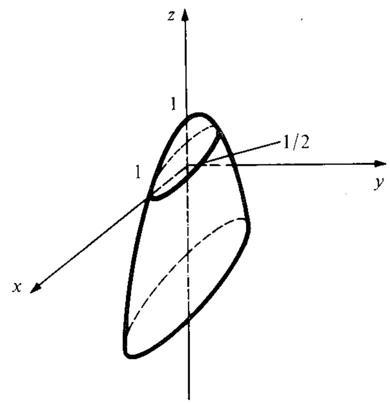
23. Elliptic hyperboloid with intercepts $(\pm 1, 0, 0)$ and $(0, \pm 1/2, 0)$.



25. Elliptic paraboloid with intercepts $(\pm 1, 0, 0)$, $(0, \pm 1/2, 0)$ and $(0, 0, -1)$.



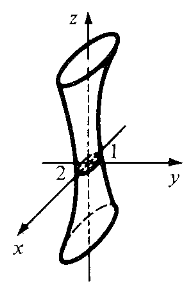
27. Elliptic paraboloid with intercepts $(\pm 1, 0, 0)$, $(0, \pm 1/2, 0)$ and $(0, 0, 1)$.

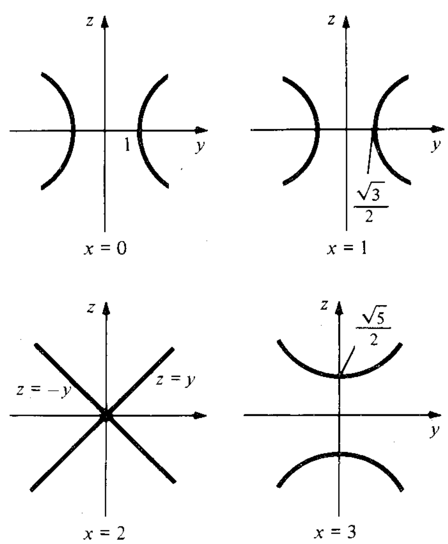


29. (a) $(x/a)^2 + (y/b)^2 = 1 + (z/c)^2$, which are ellipses.

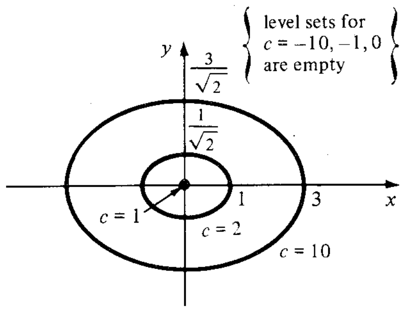
(b) x constant; $|x| < a$ gives a hyperbola opening along the y-direction, $|x| = a$ gives two lines, and $|x| > a$ gives a hyperbola opening along z-direction.

(c)



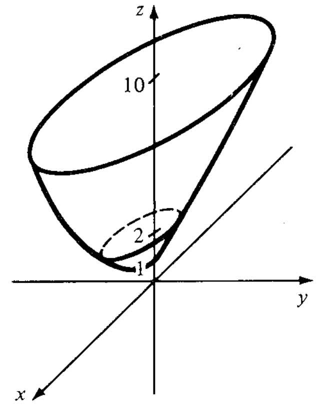


31. (a)



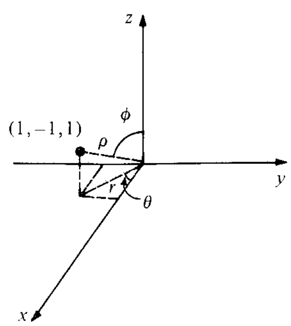
(b) For $x = \pm 1$ and $x = 2$, the equations are $z = 2y^2 + 2$ and $z = 2y^2 + 5$ which give parabolas opening upward in planes parallel to the yz-plane. For $y = \pm 1$ and $y = 2$, the equations are $z = x^2 + 3$ and $z = x^2 + 9$ which give parabolas opening upward in planes parallel to the xz-plane.

31. (c)



Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
33. $(1, -1, 1)$	$(\sqrt{2}, -\pi/4, 1)$	$(\sqrt{3}, -\pi/4, \cos^{-1}(1/\sqrt{3}))$
35. $(5\cos(\pi/12), 5\sin(\pi/12), 4)$	$(5, \pi/12, 4)$	$(\sqrt{41}, \pi/12, \cos^{-1}(4/\sqrt{41}))$
37. $(3\sqrt{6}/4, -3\sqrt{2}/4, 3\sqrt{2}/2)$	$(3\sqrt{2}/2, -\pi/6, 3\sqrt{2}/2)$	$(3, -\pi/6, \pi/4)$

33.



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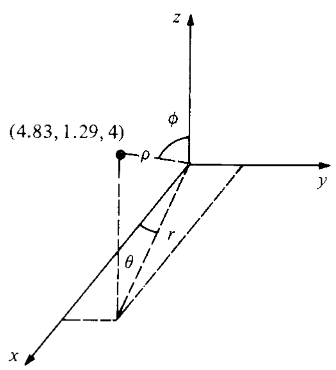

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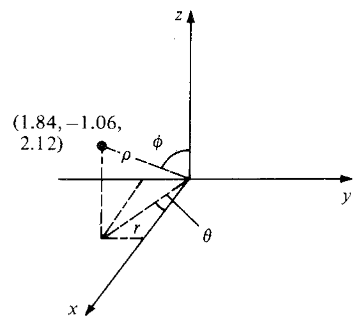
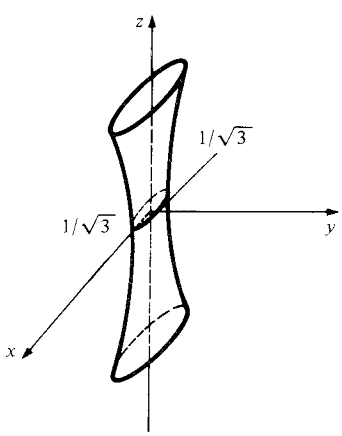


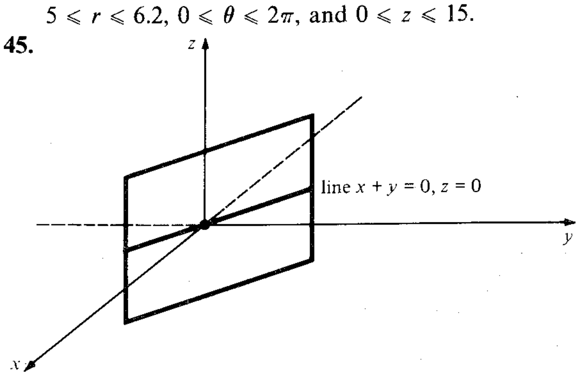
39. $3x^2 + 3y^2 = z^2 + 1$,



41. Rotate 180° around the z-axis and 90° away from the positive z-axis.

43. The rod is described by $0 \leqslant r \leqslant 5$, $0 \leqslant \theta \leqslant 2\pi$, and $0 \leqslant z \leqslant 15$. The winding is described by $5 \leqslant r \leqslant 6.2$, $0 \leqslant \theta \leqslant 2\pi$, and $0 \leqslant z \leqslant 15$.

45.



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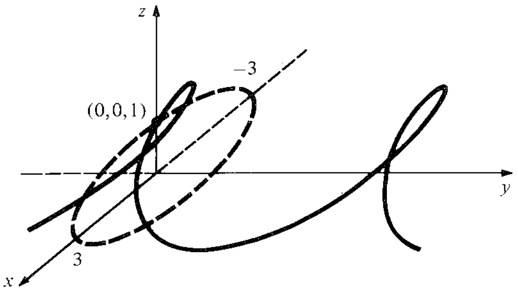

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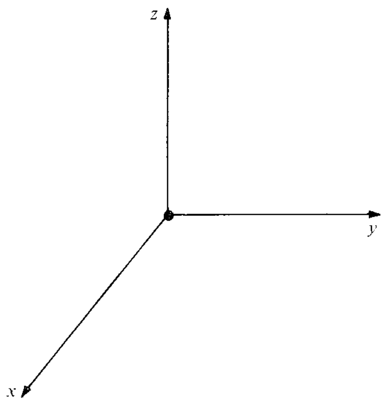
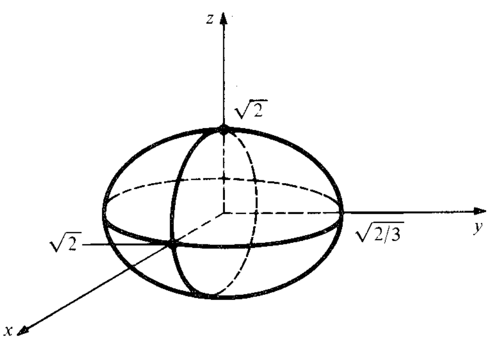
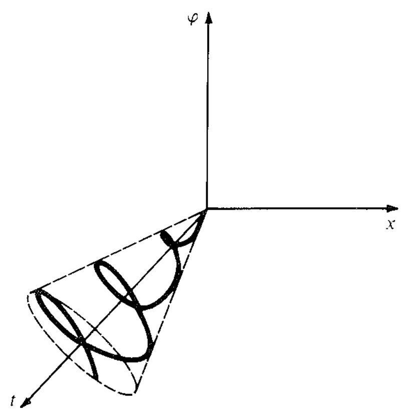


51.



53. $(x, y, z) = (2, 1/e, 0) + (t-1)(3, -1/e, -\pi/2)$
55. $e^t\mathbf{i} + \cos t\mathbf{j} - \sin t\mathbf{k}$; $e^t\mathbf{i} - \sin t\mathbf{j} - \cos t\mathbf{k}$
57. $[e^t + 2t/(1+t^2)^2]\mathbf{i} + (\cos t + 1)\mathbf{j} - \sin t\mathbf{k}$;
$[e^t + (2-6t^2)/(1+t^2)^3]\mathbf{i} - \sin t\mathbf{j} - \cos t\mathbf{k}$
59. $x = t+1,\ y = (t-1)/2,\ z = (t-2)/3$.

61. $1 + \ln 2$
63. $(9/4, -\sin(1/2) - (1/2)\cos(1/2), -2e^{1/2})$
65. (a) $x''(t) = -(k/m)x(t)$, $y''(t) = -(k/m)y(t)$, and $z''(t) = -(k/m)z(t)$.
(b) $x(t) = 0$, $y(t) = (2m/k)\sin(kt/m)$, and $z(t) = (m/k)\sin(kt/m)$.
67. $(8/81)\cos 2t/(20\sin^2 t + 16)^{3/2}$, where $x = 2\cos t$, $y = 4/3 \sin t$
69. $k = |f''(x)|/[1 + (f'(x))^2]^{3/2}$
71. $(t^4 + 3t^2 + 8)^{1/2}/(t^2+2)^{3/2}$



73. (a) The curve is $x^2 + z^2 = 2$ which can be expressed as $x = \sqrt{2}\cos t$, $y = 1$, $z = \sqrt{2}\sin t$.
(b) $(x, y, z) = (1, 1, 1) + t(-1, 0, 1)$
(c) $\int_0^{2\pi} \sqrt{(-\sqrt{2}\sin t)^2 + (0)^2 + (\sqrt{2}\cos t)^2}\, dt = 2\sqrt{2}\,\pi$.
75. (a) $\boldsymbol{\sigma}(0) = \mathbf{u}_1$, $\boldsymbol{\sigma}(\pi/2) = \mathbf{u}_2$
(b) $\boldsymbol{\sigma}(t)$ lies on the unit sphere and in the plane determined by $\mathbf{u}_1$ and $\mathbf{u}_2$
(c) $\cos^{-1}(\mathbf{u}_1 \cdot \mathbf{u}_2)$
(d) $\int_0^{\pi/2} \sqrt{1 - 2\mathbf{u}_1 \cdot \mathbf{u}_2 \sin t \cos t}\, dt$
(e) Let $\mathbf{w}$ be the unit vector in the direction of $(\mathbf{u}_1 \times \mathbf{u}_2) \times \mathbf{u}_1$. Let $\omega = \dfrac{2}{\pi}\cos^{-1}(\mathbf{u}_1 \cdot \mathbf{u}_2)$. Then $\boldsymbol{\sigma}_1(t) = \mathbf{u}_1 \cos(\omega t) + \mathbf{w}\sin(\omega t)$.

Chapter 15 Answers

15.1 Introduction to Partial Derivatives

1. $f_x = y$, $f_x(1,1) = 1$; $f_y = x$, $f_y(1,1) = 1$
3. $f_x = 1/[1 + (x-3y^2)^2]$, $f_x(1,0) = 1/2$;
$f_y = -6y/[1 + (x-3y^2)^2]$, $f_y(1,0) = 0$.
5. $f_x = ye^{xy}\sin(x+y) + e^{xy}\cos(x+y)$, $f_x(0,0) = 1$;
$f_y = xe^{xy}\sin(x+y) + e^{xy}\cos(x+y)$, $f_y(0,0) = 1$.
7. $f_x = -3x^2/(x^3+y^3)^2$, $f_x(-1,2) = -3/49$;
$f_y = -3y^2/(x^3+y^3)^2$, $f_y(-1,2) = -12/49$.
9. $f_x = yz$, $f_x(1,1,1) = 1$; $f_y = xz$, $f_y(1,1,1) = 1$;
$f_z = xy$, $f_z(1,1,1) = 1$.
11. $f_x = -y^2\sin(xy^2) + 3yze^{3xyz}$, $f_x(\pi,1,1) = 3e^{3\pi}$;
$f_y = -2xy\sin(xy^2) + 3xze^{3xyz}$, $f_y(\pi,1,1) = 3\pi e^{3\pi}$;
$f_z = 3xye^{3xyz}$, $f_z(\pi,1,1) = 3\pi e^{3\pi}$.
13. $\partial z/\partial x = 6x$; $\partial z/\partial y = 4y$.
15. $\partial z/\partial x = 2/3y + 7/3$; $\partial z/\partial y = -2x/3y^2$.
17. $\partial u/\partial x = e^{-xyz}[-yz(xy+xz+yz) + (y+z)]$;
$\partial u/\partial y = e^{-xyz}[-xz(xy+xz+yz) + (x+z)]$;
$\partial u/\partial z = e^{-xyz}[-xy(xy+xz+yz) + (x+y)]$.

19. $\partial u/\partial x = e^x\cos(yz^2)$; $\partial u/\partial y = -z^2e^x\sin(yz^2)$; $\partial u/\partial z = -2yze^x\sin(yz^2)$

21. $(xye^xe^y - xe^xe^y + xe^y + e^x)/(ye^x + 1)^2$

23. $16b(mx + b^2)^7$

25. $12 + (2/9)\cos(2/9) - 27e^2$.

27. $-4\cos(1) + 3 - 3e$

29. (a) $-x(\sin x)e^{-xy}$
(b) $0, 0, -\pi/2, -(\pi/2)e^{-\pi^2/4}$

31. 1

33. $1/6 - 3\sec^2(6)$

35. $1/7 + 3\sec^2(-15)$

37. $(tu^2)e^{stu^2}$

39. $$\frac{(-\mu\sin\lambda\mu)(1 + \lambda^2 + \mu^2) - 2\lambda\cos\lambda\mu}{(1 + \lambda^2 + \mu^2)^2}$$

41. $f_z = \lim_{\Delta z\to 0}\{[f(x, y, z + \Delta z) - f(x, y, z)]/\Delta z\}$.

43. The rate of change is approximately zero.

45. (a) $1/(1 + R_1/R_2 + R_1/R_3)^2$
(b) 36/121 times as fast.

47. $\partial^2 z/\partial x^2 = 6$, $\partial^2 z/\partial y^2 = 4$, $\partial^2 z/\partial x\partial y = \partial^2 z/\partial y\partial x = 0$

49. $\partial^2 z/\partial x^2 = 0$, $\partial^2 z/\partial y^2 = 4x/3y^3$, $\partial^2 z/\partial x\partial y = \partial^2 z/\partial y\partial x = -2/3y^2$

51. $f_{xy} = 2x + 2y$, $f_{yz} = 2z$, $f_{zx} = 0$, $f_{xyz} = 0$

53. $\partial^2 u/\partial x^2 = 24xy(x^2 - y^2)/(x^2 + y^2)^4$, $\partial^2 u/\partial y\partial x = \partial^2 u/\partial x\partial y = -6(x^4 - 6x^2y^2 + y^4)/(x^2 + y^2)^4$, $\partial^2 u/\partial y^2 = -24xy(x^2 - y^2)/(x^2 + y^2)^4$.

55. $\partial^2 u/\partial x^2 = y^4e^{-xy^2} + 12x^2y^3$, $\partial^2 u/\partial y\partial x = \partial^2 u/\partial x\partial y = e^{-xy^2}(-2y + 2xy^3) + 12x^3y^2$, $\partial^2 u/\partial y^2 = 2xe^{-xy^2}(2xy^2 - 1) + 6x^4y$.

57. Take $\delta = \varepsilon$.

59. 0

61. $52/\sqrt{13} = 4\sqrt{13}$

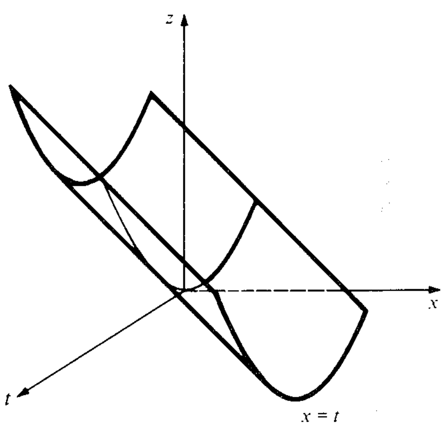
63. $-e$

65. 0

67. $g'(t_0) = -2\cos t_0\sin t_0 + 2e^{2t_0}$

69. Evaluate the derivatives and add.

71. (a) Evaluate the derivatives and compare.
(b)



73. (a) 170 units
(b) 276 units. This is the marginal productivity of capital per million dollars invested, with a labor force of 5 people and investment level of three million dollars.

75. (a) Look at the function restricted to the x-, y-, and z-axes.

75. (a) Look at the function restricted to the x-, y-, and z-axes.

77. (a) Substitute $x = 0$ into f_x.
(b) Substitute $y = 0$ into f_y to get $f_y(x, 0) = x$
(c) $f_{yx}(0,0) = \lim_{y\to 0}[(f_x(0, y) - f_x(0,0))/y]$, etc.
(d) Notice that f_x and f_y are not continuous at $(0,0)$.

15.2 Linear Approximations and Tangent Planes

1. $z = -9x + 6y - 6$.

3. $z = 1$

5. $z = 2x + 6y - 4$

7. $z = 1$

9. $z = x - y + 2$

11. $z = x + y - 1$

13. $-(1/\sqrt{3})(\mathbf{i} - \mathbf{j} - \mathbf{k})$

15. $-(1/\sqrt{3})(\mathbf{i} + \mathbf{j} - \mathbf{k})$

17. -0.415

19. -2.85

21. 1.00

23. Increasing, decreasing, increasing.

25. $1 - \Delta a/6 + \Delta v$

27. (a) 2
(b) A parabola in the yz-plane, opening upward with vertex at $(1, 0, 1)$.
(c) $(0, 1, 2)$

29. See Example 1; in this case we are dealing with the lower hemisphere.

15.3 The Chain Rule

1. $(48 + 128t)\cos(3 - 2t) - 8(3 - 2t)^2\cos(3 + 8t) + 2(3 + 8t)^2\sin(3 - 2t) + (12 - 8t)\sin(3 + 8t)$

3. $(e^{2t} - e^{-2t})(\ln(e^{2t} + e^{-2t}) + 1)$.

5. $-\sin t + 2\cos t\sin t + 3t^2$.

7. $e^{t-t^2}(1 - 2t) + e^{t^2-t^3}(2t - 3t^2) + e^{t^3-t}(3t^2 - 1)$.

9. Let $f(x, y) = x/y$.

11. $(x/\sqrt{x^2 + y^2} + 2y^2)(dx/du) + (y/\sqrt{x^2 + y^2} + 4xy)(dy/du)$.

13. $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ where $-2a - 4b + c = 0$.

15. (a) $x^x(1 + \ln x)$
(b) $x^x(1 + \ln x)$
(c) One author prefers (a), the other (b).

17. $$\frac{f'(t)g(t)h(t) + f(t)g'(t)h(t) - f(t)g(t)h'(t)}{[h(t)]^2}$$

19. 6.843

21. The half-line lies in its own tangent line. The cone in Example 6 is such a surface, as is any other surface obtained by drawing rays from the origin to the points of a space curve.

15.4 Matrix Multiplication and the Chain Rule

1. [32]

3. [44]

5. $\begin{bmatrix} \sin v & u\cos v \\ ve^{uv} & ue^{uv} \end{bmatrix}$; $\begin{bmatrix} \sin 1 & 0 \\ 1 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 10 & 13 \\ 4 & 5 \end{bmatrix}$

7. $\begin{bmatrix} yz & xz & xy \\ 1 & 1 & 1 \end{bmatrix}$; $\begin{bmatrix} 9 & 9 & 9 \\ 1 & 1 & 1 \end{bmatrix}$

11. $\begin{bmatrix} c & d \\ a & b \end{bmatrix}$

13. Undefined, the first matrix has two columns and the second matrix has three rows.

15. $\begin{bmatrix} 0 \\ b \end{bmatrix}$ **17.** $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ **19.** $\begin{bmatrix} 2 & 6 \\ 7 & 12 \end{bmatrix}$

21. $\partial z/\partial x = 26x + 6y + 70$; $\partial z/\partial y = 6x + 2y + 14$.

23. $\partial z/\partial x = \cos(3x^2 - 2y)(6x)\cos(x - 3y) + \sin(3x^2 - 2y)[-\sin(x - 3y)]$;
$\partial z/\partial y = -2\cos(3x^2 - 2y)\cos(x - 3y) + 3\sin(3x^2 - 2y)\sin(x - 3y)$.

25. (a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 2x & 2y \\ 2x & -2y \end{bmatrix}$

(b) $u = (t + s)^2 + (t - s)^2$, $v = (t + s)^2 - (t - s)^2$, $\partial(u, v)/\partial(t, s) = \begin{bmatrix} 4t & 4s \\ 4s & 4t \end{bmatrix}$

(c) Multiply the matrices in (a) and express in terms of s and t.

27. (a) $\begin{bmatrix} s & t \\ s & t \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $u = ts$, $v = -ts$, $\partial(u, v)/\partial(t, s) = \begin{bmatrix} s & t \\ -s & -t \end{bmatrix}$

(c) Multiply the matrices in (a).

29. $\partial u/\partial r = \cos\theta \sin\phi(\partial u/\partial x) + \sin\theta\sin\phi(\partial u/\partial y) + \cos\phi(\partial u/\partial z)$, $\partial u/\partial\theta = -r\sin\theta\sin\phi(\partial u/\partial x) + r\cos\theta\sin\phi(\partial u/\partial y)$,
$(\partial u/\partial\phi) = r\cos\theta\cos\phi(\partial u/\partial x) + r\sin\theta\cos\phi(\partial u/\partial y) - r\sin\phi(\partial u/\partial z)$

31. $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$.

$$\partial(r, \theta)/\partial(x, y) = \begin{bmatrix} \dfrac{x}{\sqrt{x^2 + y^2}} & \dfrac{y}{\sqrt{x^2 + y^2}} \\ \dfrac{-y}{x^2 + y^2} & \dfrac{x}{x^2 + y^2} \end{bmatrix}$$

33. $AB = \sum_{i=1}^{m} a_i \cdot (1/m) = \frac{1}{m}\sum_{i=1}^{m} a_i$, the average of the entries of A.

35. Multiply B (found in Exercise 34) by A.

37. $\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

39. Use the relations between areas, volumes, and determinants in Section 13.6.

41. (a) -16
(b) 8
(c) $-128 = -16 \cdot 8$

43.
$$\rho\begin{bmatrix} \partial^2 v_1/\partial t^2 \\ \partial^2 v_2/\partial t^2 \\ \partial^2 v_3/\partial t^2 \end{bmatrix} = (a + b)\begin{bmatrix} \partial u/\partial x \\ \partial u/\partial y \\ \partial u/\partial z \end{bmatrix} + b\begin{bmatrix} \partial^2 v_1/\partial x^2 + \partial^2 v_1/\partial y^2 + \partial^2 v_1/\partial z^2 \\ \partial^2 v_2/\partial x^2 + \partial^2 v_2/\partial y^2 + \partial^2 v_2/\partial z^2 \\ \partial^2 v_3/\partial x^2 + \partial^2 v_3/\partial y^2 + \partial^2 v_3/\partial z^2 \end{bmatrix}$$

45. (a) Substitute and use $\cos^2 + \sin^2 = 1$.
(b) Eliminate θ to find a relation between x, y, z, and φ.
(c) Look at the ratio y/x.
(d) Find $\partial(x, y, z)/\partial(u, \varphi, \theta)$ and evaluate its determinant.

47. Express $|A||B|$ as a sum of 36 terms.

49. $$\int_a^b f(x)\,dx \approx \frac{b - a}{3n}[f(x_0)\, f(x_1) \ldots f(x_n)]\begin{bmatrix} 1 \\ 4 \\ 2 \\ 4 \\ \vdots \\ 4 \\ 2 \\ 4 \\ 1 \end{bmatrix}$$

Review Exercises for Chapter 15

1. $g_x = \pi\cos(\pi x)/(1 + y^2)$;
$g_y = -2y\sin(\pi x)/(1 + y^2)^2$

3. $k_x = z^2 + z^3\sin(xz^3)$; $k_z = 2xz + 3xz^2\sin(xz^3)$.

5. $h_x = z$; $h_y = 2y + z$; $h_z = x + y$

7. $f_x = -[\cos(xy) + y\sin(xy)]/[e^x + \cos(xy)]$;
$f_y = -x\sin(xy)/[e^x + \cos(xy)]$; $f_z = 0$.

9. $g_x = z + x^2e^{x+z}$; $g_y = 0$; $g_z = x + e^z\int_0^x t^2e^t\,dt$

11. $g_{xy} = g_{yx} = -2\pi y\cos(\pi x)/(1 + y^2)^2$

13. $k_{xz} = k_{zx} = 2z + 3z^2\sin(xz^3) + 3xz^5\cos(xz^3)$

15. $h_{xz} = h_{zx} = 1$ **17.** 1 **19.** $-\sin(2)$

21. (a) 35.25 minutes
(b) $\partial T/\partial x|_{(27,65)} = -0.598$ minutes/foot; this means that in diving from 27 to 28 feet, your time decreases about 36 seconds. $\partial T/\partial V|_{(27)} = 0.542$ minutes/cubic foot; this means that bringing an extra cubic foot of air will give you about 33 seconds more diving time.

23. 4

25. 0

27. $z = 2x + 2y - 2$

29. $z = 1$

31. 9.00733

33. 0.999

35. 5.002

37. $t = \sqrt{14}(-3 + 2\sqrt{709})/70$

39. $d[f(\sigma(t))]/dt = 2t/[(1 + t^2 + 2\cos^2 t)(2 - 2t^2 + t^4)] - 4t(t^2 - 1)\ln(1 + t^2 + 2\cos^2 t)/(2 - 2t^2 + t^4)^2 - 4\cos t\sin t/[(1 + t^2 + 2\cos^2 t)(2 - 2t^2 + t^4)]$

41. (a) Use the chain rule with $x - ct$ as intermediate variable.
(b) It shifts with velocity c along the x axis, without changing its shape.

43. The radius is increasing by 15 cm/hr.

45. $[f'(t)g(t) + f(t)g'(t)]\exp[f(t)g(t)]$

47. $(1 + 2y - 2x)\exp(x + 2xy)$

49. $y = -x/6 + 7/12$

51. [4]

53. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **55.** $\begin{bmatrix} 6 & -3 \\ 12 & -6 \end{bmatrix}$

57. $\begin{bmatrix} 0 & -1 & 4 \\ -3 & -2 & -1 \\ 3 & 1 & 5 \end{bmatrix}$

59. $\begin{bmatrix} 3 & 1 & -3 \\ 5 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

61. $\partial z/\partial x = 4(e^{-2x-2y+2xy})(1+y)/(e^{-2x-2y} - e^{2xy})^2$,
$\partial z/\partial y = 4(e^{-2y-2x+2xy})(1+x)/(e^{-2x-2y} - e^{2xy})^2$

63. $\partial z/\partial u = \partial z/\partial x + \partial z/\partial y$;
$\partial z/\partial v = \partial z/\partial x - \partial z/\partial y$.

65. (a) $n = PV/RT$; $P = nRT/V$; $T = PV/nR$; $V = nRT/P$
(b) $\partial P/\partial T$ represents the ratio between the change ΔP in pressure and the change ΔT in temperature when the volume and number of moles of gas are held fixed.
(c) $\partial V/\partial T = nR/P$; $\partial T/\partial P = V/nR$; $\partial P/\partial V = -nRT/V^2$. Multiply, remembering that $PV = nRT$.

67. (a) One may solve for any of the variables in terms of the other two.
(b) $\partial T/\partial P = (V-\beta)/R$;
$\partial P/\partial V = -RT/(V-\beta)^2 + 2\alpha/V^3$;
$\partial V/\partial T = R/[(V-\beta)(RT/(V-\beta)^2 - 2\alpha/V^3)]$
(c) Multiply and cancel factors.

69. Notice that $y = x^2$, so if y is constant, x cannot be a variable.

71. $\partial^3 u/\partial x\partial y\partial z = \partial^3 u/\partial y\partial x\partial z = \partial^3 u/\partial y\partial z\partial x$

73. Differentiate and substitute.

75. Use the chain rule.

77. Yes. The second partial derivatives are not continuous at the origin; the graph has a 'crinkle' at the origin.

Chapter 16 Answers

16.1 Gradients and Directional Derivatives

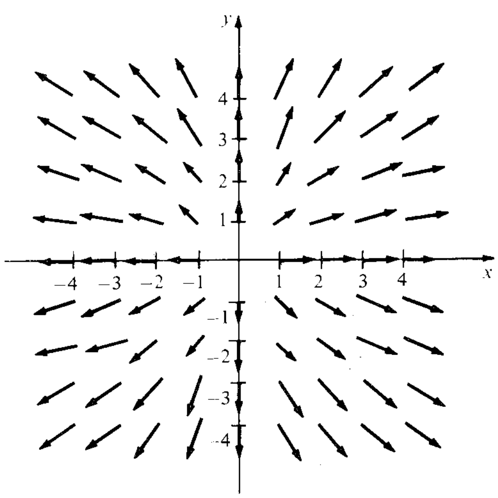
1. $(x/\sqrt{x^2+y^2+z^2})\mathbf{i} + (y/\sqrt{x^2+y^2+z^2})\mathbf{j} + (z/\sqrt{x^2+y^2+z^2})\mathbf{k}$

3. $\mathbf{i} + 2y\mathbf{j} + 3z^2\mathbf{k}$

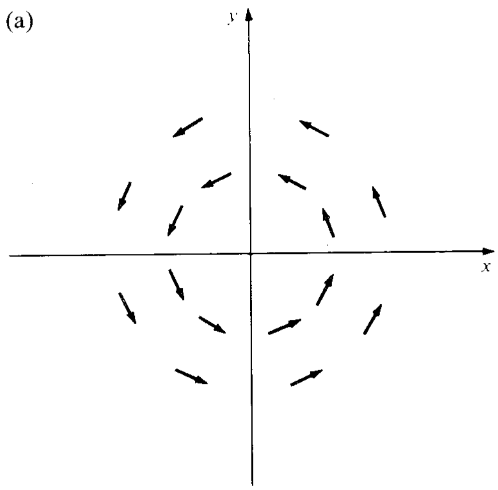
5. $[x/(x^2+y^2)]\mathbf{i} + [y/(x^2+y^2)]\mathbf{j}$

7. $(1+2x^2)\exp(x^2+y^2)\mathbf{i} + 2xy\exp(x^2+y^2)\mathbf{j}$.

9.



11. (a)



(b) $\partial(-y)/\partial y \neq \partial(x)/\partial x$

13. $\dfrac{\partial}{\partial x}\left(\dfrac{1}{x^2+y^2+z^2}\right) = \dfrac{-2x}{r^4}$, etc.

15. $2e^t\cos t + \cos^2 t - \sin^2 t$

17. $t/\sqrt{1+t^2}$

19. The angle between the gradient and the velocity vector is between 0 and $\pi/2$.

21. $-11 - 16\sqrt{3}$

23. $17/\sqrt{2}$

25. $-14/\sqrt{3}$

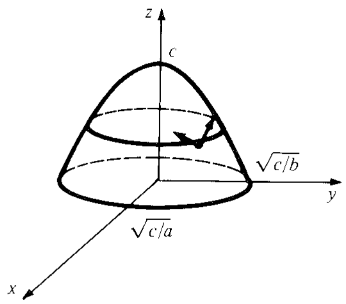
27. $\pi/8 - 1/2$

29. $(\mathbf{i} + 2\mathbf{j})/\sqrt{5}$

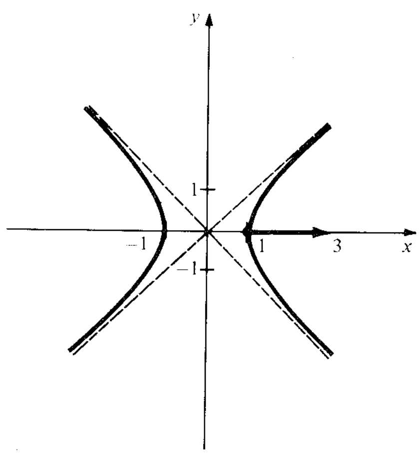
31. $e[(\sin 1)\mathbf{i} + (\cos 1)\mathbf{j}]$

33. (a) $(1, 2, 3)$
(b) $-2\sqrt{14}\,e^2$ degrees per second
(c) She should fly outside the cone with vertex at $(1,1,1)$, axis along $(1,2,3)$ and sides at an angle of $\pi/3$ from the axis.

35. $\mathbf{d}_1 = [-(0.03 + 2by_1)/2a]\mathbf{i} + y_1\mathbf{j}$, $\mathbf{d}_2 = [-(0.03 + 2by_2)/2a]\mathbf{i} + y_2\mathbf{j}$ where y_1 and y_2 are the solutions of $(a^2 + b^2)y^2 + 0.03by + \left(\dfrac{0.03^2}{4} - a^2\right) = 0$.



37. $2\mathbf{i}$



39. $(1/\sqrt{3})(\mathbf{i} + \mathbf{j} + \mathbf{k})$

41. Write out each expression in terms of partial derivatives and use the properties of differentiation.

43. (a) $(1/\sqrt{2}, 1/\sqrt{2})$
(b) The directional derivative is 0 in the direction $(x_0\mathbf{i} + y_0\mathbf{j})/\sqrt{x_0^2 + y_0^2}$.
(c) The level curve through (x_0, y_0) must be tangent to the line through $(0, 0)$ and (x_0, y_0). The level curves are lines or half lines emanating from the origin.

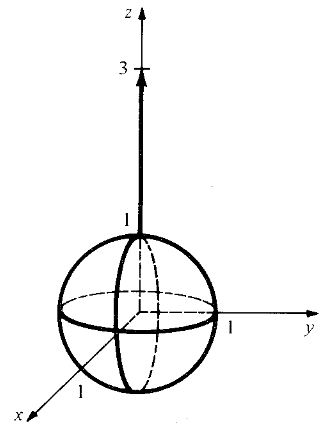
45. $\nabla f(1, 3) = (2, -2)$; $-2/\sqrt{13}$

47. (a) $\dfrac{\lambda}{2\pi\varepsilon_0}\left\{\left(\dfrac{x + x_0}{r_1^2} - \dfrac{x - x_0}{r_2^2}\right)\mathbf{i} + 2y\left(\dfrac{1}{r_1^2} - \dfrac{1}{r_2^2}\right)\mathbf{j}\right\}$
(b) Compute the indicated partial derivatives.

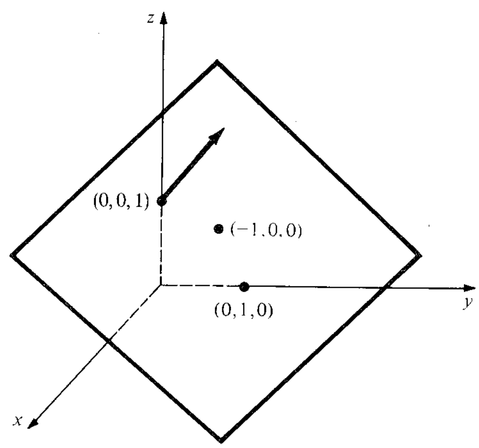
49. The function f must satisfy Laplace's equation: $f_{xx} + f_{yy} = 0$.

16.2 Gradients, Level Surfaces, and Implicit Differentiation

1. $\nabla f(0, 0, 1) = 2\mathbf{k}$



3. $\nabla f(0, 0, 1) = -\mathbf{i} + \mathbf{j} + \mathbf{k}$



5. $(1/\sqrt{129})(8\mathbf{i} + 8\mathbf{j} + \mathbf{k})$

7. $\mathbf{k}$

9. $V = Qq/r$; the level surfaces are spheres, which are orthogonal to radial vectors.

11. $x + 2\sqrt{3}\,y + 3z = 10$.

13. $3x + 8y + 3z = 20$

15. $x + y + z = 3$

17. $x + 2y - 3 = 0$

19. $x + y - \pi/2 = 0$

21. $(1, 1, 1) + t(1, 1, 1)$

23. $(1, 1, 1) + t(1, -1, -1)$

25. $-x/2y$

27. $y/x = 1/10$

29. $3x^2/(\cos y - 4y^3)$

31. $1/2$

33. $-1 - 2\sqrt{3}/3$

35. At $(0, 0)$, the slope of $y = \sqrt{x}$ is infinite.

37. At $(0, 0)$, the slope of $y = x^{1/5}$ is infinite.

39. $dx/dt = (-1/y)(dy/dt)$

41. $x^3(dx/dt) + y^3(dy/dt) = 0$

43. (a) $dx/dy = -(\partial z/\partial y)/(\partial z/\partial x)$
(b) $(\cos y - 4y^3)/3x^2$;
$-y(2e^{x+y^2} + 3y)/e^{x+y^2}$

45. $(1/x - 1)(dx/dt) + (-\tan y)(dy/dt) = 0$

47. (a) $z = 2x - 4y - 5$
(b) The slope is the tangent of the angle between the upward pointing unit normal vector and the z-axis. The slope in this case is $2\sqrt{5}$.

49. Crosses at $(2,2,0)$, $\sqrt{5}/10$ seconds later.

51. (a) They are perpendicular.
(b) If it were not equipotential, there would be places where the force of gravity is not perpendicular to the surface and the water would flow to correct this. The rotation of the earth and tides (among other things) spoil the approximation.

16.3 Maxima and Minima

1. Local minimum at $(1,0)$, local maximum at $(-1,0)$

3. $(0,0)$ is a local minimum.

5. $(0,0)$ is a local maximum.

7. $\sqrt{3/2}$

9. The height is $4b^{2/3}/s^{2/3}$.

11. Minimum

13. Saddle point

15. Minimum (although the test per se is inconclusive)

17. $(-3,2)$, minimum

19. $(0,0)$, neither

21. $(3,7)$, minimum

23. $(1,1)$, minimum

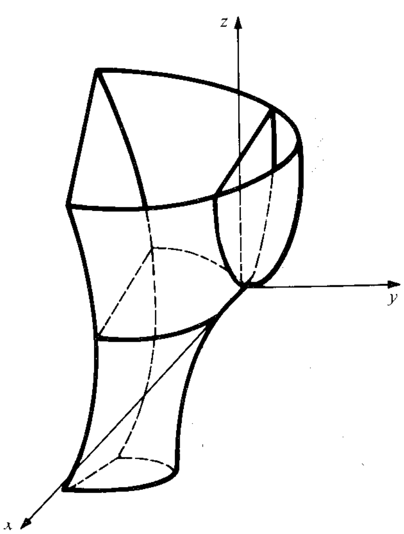
25. $(4/5, -9/10)$, minimum

27. $(0,0)$, neither

29. $(0,0)$, neither

31. $(0,0)$ is a saddle point.

33. The second derivative test fails, but from the accompanying graph, we can see that $(0,0)$ is neither a local maximum nor minimum.



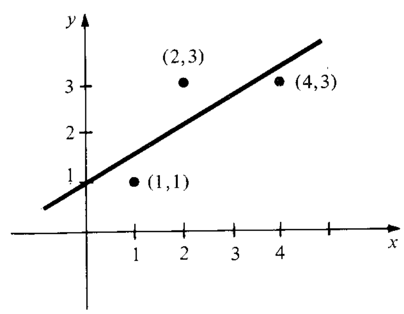
35. (a) Calculate $\partial z/\partial x$ and $\partial z/\partial y$ and set them equal to zero.
(b) The maximum is at $(0,0)$ and local maxima [resp. minima] occur on circles of radius $r_2, r_4, \ldots$ [resp. $r_1, r_3, \ldots$] where $0 < r_1 < r_2 < r_3 < \cdots$ are the solutions of $\pi r = \tan(\pi r)$.
(c) Symmetric in every vertical plane through the origin and under any rotation about the z-axis.

37. (a) Set $\partial w/\partial p_i = 0$. This occurs when $T_{i-1}/T_i = (p_i^2/(p_{i-1}p_{i+1}))^{1-(1/n)}$.
(b)
$$p_1 = \left[\left(\frac{T_0^3}{T_1T_2T_3}\right)^{n/(n-1)} p_0^3 p_4\right]^{1/4}$$
$$p_2 = \left[\left(\frac{T_0T_1}{T_2T_3}\right)^{n/(n-1)} p_0 p_4\right]^{1/2}$$
$$p_3 = \left[\left(\frac{T_0T_1T_2}{T_3^3}\right)^{n/(n-1)} p_0 p_4^3\right]^{1/4}$$

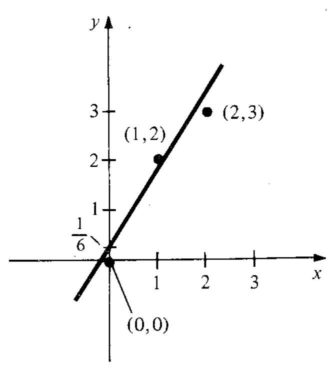
39. $A = 2$, $B = 1$, $C = 2$ so $A > 0$ and $AC - B^2 = 3 > 0$. Thus the point is a local minimum.

41. (a) $(0,0)$ is a saddle point.
(b) The behavior changes qualitatively at $C = \pm 2$. For $-2 < C < 2$, $(0,0)$ is a strict minimum; for $C < -2$ or $C > 2$, $(0,0)$ is a saddle point. For $C = \pm 2$, $(0,0)$ is a minimum.

43. (a) $b = 1$, $m = 4/7$



(b) $b = 1/6$, $m = 3/2$

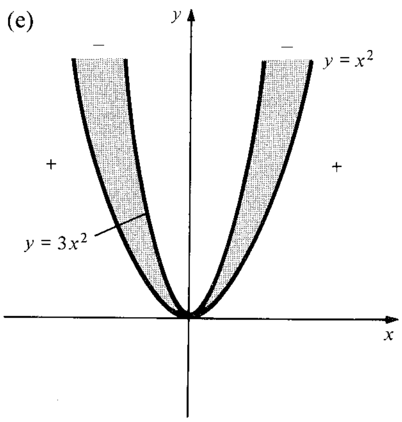


45. $\dfrac{\partial s}{\partial b} = -2\sum(y_i - mx_i - b)$ and
$\dfrac{\partial s}{\partial m} = -2\sum x_i(y_i - mx_i - b)$; set these equal to zero and use properties of summation (Section 4.1).

47. Compute $\partial^2 s/\partial m^2$, $\partial^2 s/\partial m \partial b$, and $\partial^2 s/\partial b^2$ directly.

49. (a) $e = \sqrt{B^2 - CA}/A$
(b) $y = Ax/(Ae - B)$, $y = -Ax/(B + Ae)$
(c) $g(x, y)$ is positive when (i) $y > Ax/(Ae - B)$ and $y > -Ax/(B + Ae)$ or
(ii) $y < Ax/(Ae - B)$ and $y < -Ax/(B + Ae)$
(d) If $A = 0$ and $B = 0$, then $AC - B^2 = 0$. Thus B cannot be zero if $A = 0$ and $AC - B^2 < 0$. Rewrite g as $g(x, y) = y(2Bx + Cy)$. Note that $y = 0$ and $2Bx + Cy = 0$ are two lines intersecting at the origin. Thus $g(x) > 0$ in the region above $2Bx + Cy = 0$ and the negative x-axis, and the region below $2Bx + Cy = 0$ and the positive x-axis.

51. (a) $f_x(0, 0) = 0$, $f_y(0, 0) = 0$.
(b) Use one variable calculus on h
(c) $f > 0$ if $y > 3x^2$ or $y < x^2$ and $f < 0$ if $x^2 < y < 3x^2$.
(d) $f(x, y) = 0$ if $y = x^2$ or $y = 3x^2$.
(e)



(f) The segment on which h is positive shrinks to 0 as $\theta \to 0$.

53. $(0, 0, 0)$ is closest for $a \le 1/8$; $(\pm\sqrt{(8a - 1)/32},\ 0,\ (8a - 1)/8)$ are closest for $a > 1/8$.

55. Let $d^2 = (x - a)^2 + (y - b)^2 + (k(x, y) - c)^2$ and set $\partial(d^2)/\partial x$ and $\partial(d^2)/\partial y$ equal to zero.

16.4 Constrained Extrema and Lagrange Multipliers

1. The minimum value is 0 (occurs at $(0, 0)$), maximum value is 3 (occurs at $(0, 1)$ and $(0, -1)$).

3. The minimum value is 8 (occurs at $(0, 1)$ and $(0, -1)$) maximum value is 15 (occurs at $(1, 0)$ and $(-1, 0)$).

5. $\sqrt{35/2}$ is the maximum value, $-\sqrt{35/2}$ is the minimum value. There are no interior critical points.

7. $\sqrt{2}$ is the maximum value, $-\sqrt{2}$ is the minimum value. $(\pm\sqrt{1/2}, \pm\sqrt{1/2})$ are the critical points.

9. $1/4$ is the maximum value. $(1/2, 1/2)$ is the critical point.

11. $\sqrt{10}$ is the maximum value, $-\sqrt{10}$ is the minimum value. $(\pm\sqrt{10}/10, \mp 3\sqrt{10}/10)$ are the critical points.

13. $x = y = 25{,}000$; $z = 50{,}000$.

15. Horizontal length is $\sqrt{qA/p}$, vertical length is $\sqrt{pA/q}$.

17. $(Q_2/Q_1)^{1/3}$

19. (a) $(\sqrt{2}/2, \sqrt{2}/2, 3/2)$ and $(-\sqrt{2}/2, -\sqrt{2}/2, 3/2)$ are maxima, while $(-\sqrt{2}/2, \sqrt{2}/2, 1/2)$, and $(\sqrt{2}/2, -\sqrt{2}/2, 1/2)$ are minima.
(b) h is increasing at $(\pm 1, 0)$ and decreasing at $(0, \pm 1)$.
(c) $(\sqrt{2}/2, \sqrt{2}/2, 3/2)$, $(-\sqrt{2}/2, -\sqrt{2}/2, 3/2)$ are maxima, $(0, 0, 0)$ is the minimum.

21. (a) $C = \dfrac{\rho\pi n_1^2 i_1^2}{\alpha h}\left(\dfrac{D_1}{x} + \dfrac{D_2}{y}\right)$
(b) $x = \dfrac{D_2 - D_1}{2\left(1 + \sqrt{D_2/D_1}\right)}$,
$y = \dfrac{D_2 - D_1}{2\left(\sqrt{D_1/D_2} + 1\right)}$

23. (a) ∇f is parallel to ∇g.
(b) The maximum value of $\sqrt{3}/9$ occurs at $(\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)$, $(\sqrt{3}/3, -\sqrt{3}/3, -\sqrt{3}/3)$, $(-\sqrt{3}/3, -\sqrt{3}/3, \sqrt{3}/3)$ and $(-\sqrt{3}/3, \sqrt{3}/3, -\sqrt{3}/3)$. The minimum value of $-\sqrt{3}/9$ occurs at $(-\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)$, $(\sqrt{3}/3, -\sqrt{3}/3, \sqrt{3}/3)$, $(\sqrt{3}/3, \sqrt{3}/3, -\sqrt{3}/3)$ and $(-\sqrt{3}/3, -\sqrt{3}/3, -\sqrt{3}/3)$.
(c) $x = y = 8$, $h = 4$.

Review Exercises for Chapter 16

1. $[y\exp(xy) - y\sin(xy)]\mathbf{i} + [x\exp(xy) - x\sin(xy)]\mathbf{j}$

3. $[2x\exp(x^2) + y^2\sin(xy^2)]\mathbf{i} + 2xy\sin(xy^2)\mathbf{j}$

5. (a) $(9/\sqrt{2})\cos(3)$
(b) $(\mathbf{i} - 2\mathbf{j})/\sqrt{5}$

7. (a) $\sqrt{2}/e$
(b) $(-\mathbf{i} - 2\mathbf{j})/\sqrt{5}$

9. $3x + 4y - z = 4$

11. $x + y + z = \sqrt{3}$

13. $(2x + y)(dx/dt) + (x + 2y)(dy/dt) = 0$

15. $(x^2 + y^2)(dx/dt) + 2xy(dy/dt) = 0$

17. 1

19. 1

21. $(0, 0)$ is a saddle point.

23. $(0, 0)$ is a saddle point.

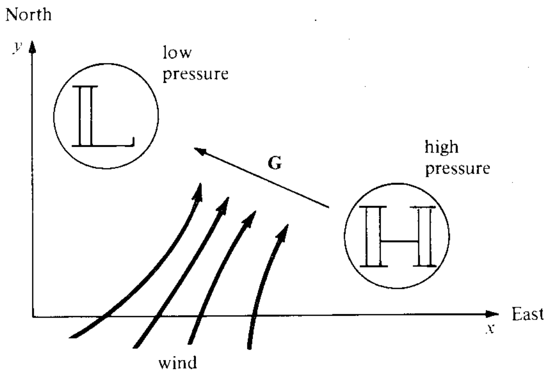
25. $(-1, 0)$ is a saddle point, $(0, 0)$ a local maximum, and $(2, 0)$ a local minimum.

27. $(n, 0)$, n an integer, are saddle points.

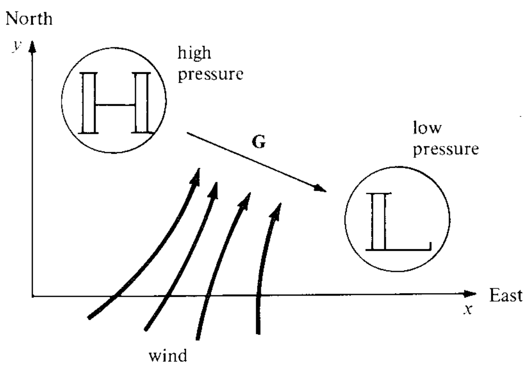
29. (a) $\|\mathbf{G}\| = ((\partial P/\partial x)^2 + (\partial P/\partial y)^2)^{1/2}$

(b) According to Newton's second law of motion, **G** creates a force on the air mass which produces a proportionate acceleration in the direction of **G**.

(c)



(d) If, in the Southern Hemisphere, you stand with your back to the wind, the high pressure is on your left and the low pressure is on your right.



31. (a) $(\pm 1, 0, 0)$ are closest, distance is 1.
(b) $(-\sqrt{41}/6, \sqrt{41}/6, 1/6)$,
$(\sqrt{41}/6, -\sqrt{41}/6, 1/6)$ are closest,
the distance is $\sqrt{83}/6$.
(c) $(-1,-1,1)$, $(-1,1,-1)$, $(1,-1,-1)$ and $(1,1,1)$ are closest, the distance is $\sqrt{3}$.

33. 0.382 is the minimum value, 2.618 is the maximum value.

35. 0.540 is the minimum value, 1 is the maximum value.

37. The partials of $k(x, y, \lambda) = x + 2y\sec\theta + \lambda(xy + y^2\tan\theta - A)$ must all vanish

39. (a) Use the second derivative test.
(b) Since the maximum and minimum must occur on the boundary (by (a)), both are zero. Hence f is everywhere zero.

41. (a) $(1,1,2)$, $x + y + 2z = 6$;
$(2,3,2)$, $2x + 3y + 2z = 9$.
(b) $\theta = 0.47$
(c) $(1,1,2) + t(-4,2,1)$

43. (a) A normal vector to the tangent plane is $f_x\mathbf{i} + f_y\mathbf{j} - \mathbf{k}$.
(b) The slope of the plane relative to the xy plane is 5; the plane contains the line through the point $(1,0,2)$ parallel to the y-axis.

45. Equate the four partial derivatives equal to zero. Eliminate λ_1 by subtracting two equations and λ_2 by dividing two equations.

47. 600

49. Approximately 570

51. $(4,2)$

53. (a) $dy/dx = -(2x)/(3y^2 + e^y)$
(b)

$$dy_1/dx = \frac{(\partial F_1/\partial y_2)(\partial F_2/\partial x) - (\partial F_2/\partial y_2)(\partial F_1/\partial x)}{(\partial F_2/\partial y_2)(\partial F_1/\partial y_1) - (\partial F_2/\partial y_1)(\partial F_1/\partial y_2)}$$

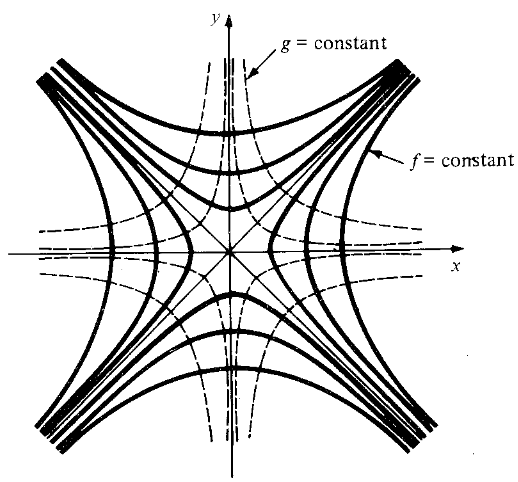
$$dy_2/dx = \frac{(\partial F_1/\partial y_1)(\partial F_2/\partial x) - (\partial F_1/\partial x)(\partial F_2/\partial y_1)}{(\partial F_1/\partial y_2)(\partial F_2/\partial y_1) - (\partial F_2/\partial y_2)(\partial F_1/\partial y_1)}$$

(c) $dy_1/dx = -(2x + \sin x)/2y_1$,
$dy_2/dx = (\cos x - 2x)/y_2$.

55. (a) Use equality of mixed partials of $f(x, y)$
(b) Use equality of mixed partials of $f(x, y, z)$
(c) No

57. (a) $x^2\exp(xy^2) + C$
(b) No function exists.
(c) $\ln(1 + x^2 + y^2) + C$.
(d) No function exists.

59. (a) $(x_0, -y_0)$
(b) (y_0, x_0)
(c) $g(x, y) = xy$
(d) Since ∇g is normal to the level curves of g, the tangent to these curves is normal to level curves of f.
(e)



61. (a) $f_x = -2xy^3/(x^2 + y^2)^2$,
$f_y = y^2(3x^2 + y^2)/(x^2 + y^2)^2$. $f_x(0,0) = 0$ and $f_y(0,0) = 1$ (compute the latter two using limits).
(b) $(\partial/\partial r)f(r\cos\theta, r\sin\theta) = \cos^2\theta(-2\sin^3\theta + 3\sin^2\theta + 1)$ is defined for all θ.
(c) $\nabla f(0,0)\cdot(\cos\theta, \sin\theta) = \sin\theta$ disagrees with the formula in (b). There is no contradiction because the partial derivatives are not continuous at $(0,0)$ (see Exercise 20, p. 783).

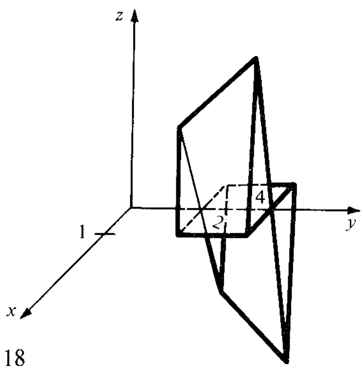
Chapter 17 Answers

17.1 The Double Integral and Iterated Integral

1. 12

3. (a) Divide D into $D_1 = [-1, 0] \times [2, 3]$, $D_2 = [-1, 0] \times (3, 4]$, *and* $D_3 = (0, 1] \times [2, 4]$. Let $g(x, y)$ be -4 on D_1, -5 on D_2, and 0 on D_3.

(b) The part of the integral for $x \leqslant 0$ is the negative of the part for $x \geqslant 0$.



5. 18

7. 16/9

9. $e/2 - 1/2e$

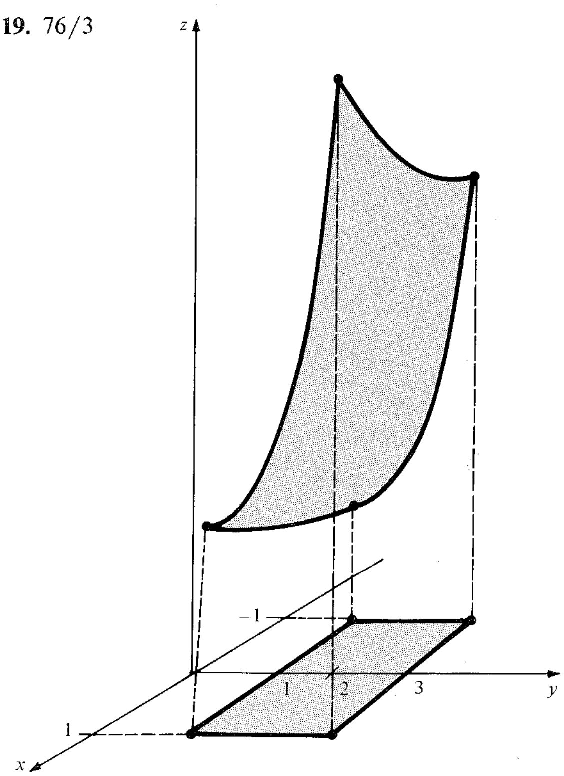
11. 50

13. 4/3

15. $45/4 + (15/2)\ln(3/2)$

17. 0; this agrees with the answer in Exercise 3(b).

19. 76/3



21. 25/6 grams.

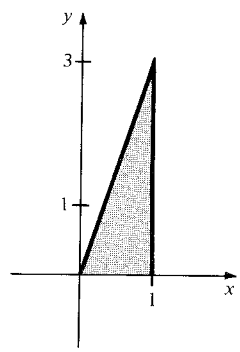
23. Use partition points obtained from subrectangles for both the functions being added.

25. (a) $0.88[1 - \sin^2\alpha \cos^2(2\pi T/365)]^{1/2}\cos(2\pi t/24)] + 0.67 \sin\alpha \cos(2\pi T/365)$, where $\alpha = 23.5°$

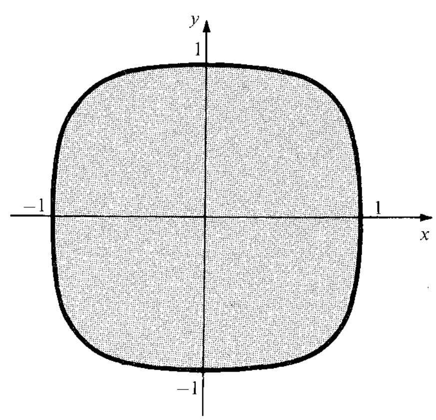
(b) This is the total solar energy received in the state between times t_1 and t_2 on day T.

17.2 The Double Integral Over General Regions

1. Both a type 1 and a type 2 region.



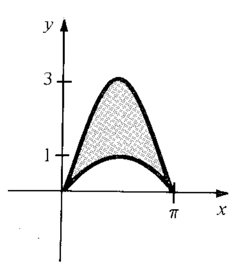
3. Both a type 1 and a type 2 region.



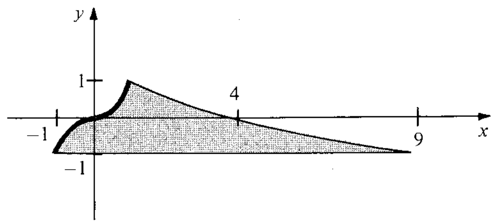
5. 7/12

7. 64/35

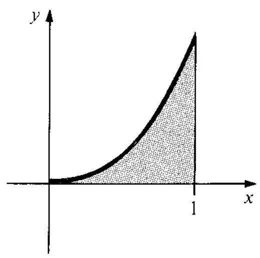
9. Type 1; $2\pi + \pi^2$



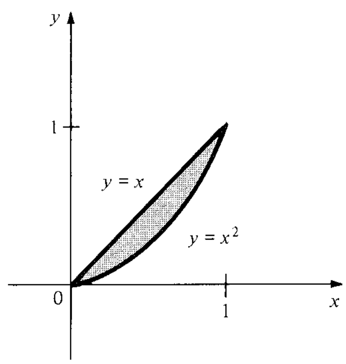
11. Type 2; 104/45



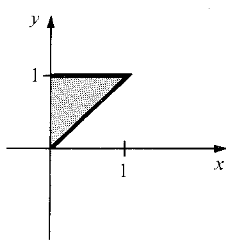
13. Type 1; 33/140



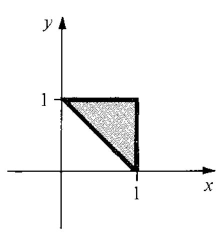
15. Type 1; 71/420



17. 1/8



19. 7/12

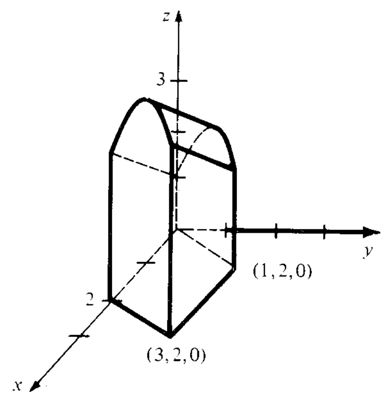


21. 1/3
23. 19/3

25. The result of the first integration is the length of a section of the region.
27. Type 1 states include: Wyoming, Colorado, Nevada, New Mexico, Kansas, and Ohio. Type 2 states include: Wyoming, Colorado, Kansas, North Dakota, and Vermont.

17.3 Applications of the Double Integral

1. 12π
3. $(4/5)(2 + 9\sqrt[3]{4})$
5. $10 + 8/\pi$



7. $(16e^2 - 16 + \pi^4)/32$	**9.** 243/80
11. $[\pi^2 - \sin(\pi^2)]/\pi^3$	**13.** 2
15. 2/3	**17.** $2a^2/3$
19. (11/18, 65/126)	**21.** (7/5, 0)
23. $(4/15)(9\sqrt{3} - 8\sqrt{2} + 1)$	**25.** $\sqrt{2}\,\pi/4$

27. Compare the formulas for average value and center of mass.
29. (a) Write $z = \pm\sqrt{r^2 - x^2 - y^2}$ over the region $0 \leqslant x \leqslant r,\ -\sqrt{r^2 - x^2} \leqslant y \leqslant \sqrt{r^2 - x^2}$.
(b) The result is independent of r.
31. \$503.64
33. $2\int_a^b \int_{-f(x)}^{f(x)} \sqrt{[f(x)]^2 - y^2}\, dy\, dx = \pi \int_a^b [f(x)]^2\, dx.$
(This is the disk method; see p. 423.)

17.4 Triple Integrals

1. 7	**3.** −8	**5.** Type I
7. Type I	**9.** $25\sqrt{2/3}\,\pi$	**11.** 1/2
13. 0	**15.** $a^5/20$	**17.** 0

19. 3/10
21. The double integral of F over the base of the box times the height of the box.
23. Interchange x and z in Example 4.
25. The region under the graph is of type I.
27. (a) Use iterated integrals and the constant multiple rule for definite integrals.
(b) $e^{x+y+z} = e^x e^y e^z$.

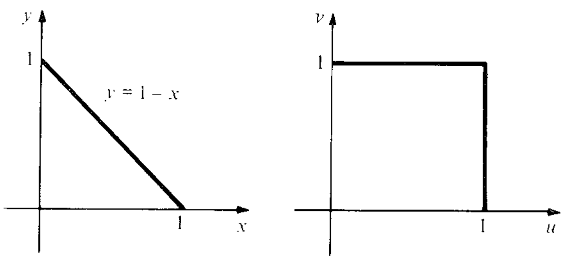
17.5 Integrals in Polar, Cylindrical, and Spherical Coordinates

1. $64\pi/5$
3. $\sqrt{\pi/10}$
5. 128π
7. $5\pi(e^4 - 1)/2$
9. $2\pi[\sqrt{2} - \ln(1 + \sqrt{2})]$
11. $4\pi\ln(a/b)$
13. $(\pi/6)(8 - 3\sqrt{3})$
15. $4\pi\sqrt{6}$
17. $1/\sqrt{\pi\sigma}$
19. (a) Use the linear approximations

$$\Delta x \approx \frac{\partial x}{\partial u}\Delta u + \frac{\partial x}{\partial v}\Delta v \text{ and } \Delta y \approx \frac{\partial y}{\partial u}\Delta u + \frac{\partial y}{\partial v}\Delta v.$$

(b) Use the fact that $\left|\dfrac{\partial(x, y)}{\partial(r, \theta)}\right| = r.$

21. The conditions $0 \leqslant y \leqslant 1 - x$ and $0 \leqslant x \leqslant 1$ are equivalent to $0 \leqslant x + y \leqslant 1$ and $0 \leqslant \dfrac{y}{x + y} \leqslant 1$.

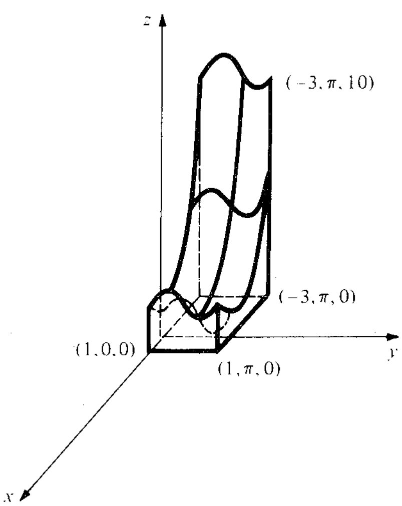


17.6 Applications of Triple Integration

1. (a) ρ, where ρ is the (constant) mass density.
(b) $41/3$
3. $(1/2, 1/2, 1/2)$
5. $\frac{\pi}{4}$ (as in Example 2)
7. (a) kc^2
(b) Along the sphere $x^2 + y^2 + z^2 = c^2$
9. $1/4$
11. Letting d be density, the moment of inertia is $d\int_0^k\int_0^{2\pi}\int_0^{a\sec\phi}\rho^4\sin^3\phi\,d\rho\,d\theta\,d\phi$
13. $1.00 \times 10^8(\mathrm{m/s})^2$
15. (a) The only plane of symmetry for the body of an automobile is the one dividing the left and right sides of the car.
(b) $\bar{z}\cdot\int\int\int_W \rho(x, y, z)\,dx\,dy\,dz$ is the z-coordinate of the center of mass times the mass of W. Rearrangement of the formula for $\bar{z}$ gives the first line of the equation. The next step is justified by the additivity property of integrals (see rule 2 on p. 841). By symmetry, we can replace z by $-z$ and integrate in the region above the xy-plane. Finally, we can factor the minus sign outside the second integral and since $\rho(x, y, z) = \rho(u, v, -w)$, we are subtracting the second integral from itself. Thus, the answer is 0.
(c) In part (b), we showed that $\bar{z}$ times the mass of W is 0. Since the mass must be positive, $\bar{z}$ must be 0.
(d) By part (c), the center of mass must lie in both planes.
17. Follow the pattern on p. 880 for the case of constant density. Be sure to use different symbols for the density and the spherical coordinate $\sqrt{x^2 + y^2 + z^2}$.

Review Exercises for Chapter 17

1. $960 - \sin(11) + \sin(10) + \sin(7) - \sin(6) \approx 961.4$
3. $15/2$
5. $64/3$
7. $\pi\ln(\sec 1 + \tan 1)$
9. $1/48$
11. $10/3$
13. $4\pi abc/3$
15. $2\pi(1 - 1/\sqrt{a + 1})/3$
17. $(25 + 10\sqrt{5})\pi/3$
19. $(4\pi/3)(1 - \sqrt{2}/2)$
21. $40\pi/3$



23. $10/3$
25. Cut with the planes $x + y + z = \sqrt[3]{k/n}$, $1 \leqslant k \leqslant n - 1$, k an integer.
27. Use the formula for surface area.

29. $(0, 0, -0.203)$
31. $2\pi(2\sqrt{2} - 1)/3$
33. $\pi(5\sqrt{5} - 1)/6$
35. $\pi^2/8$
37. $(\pi/4)\ln(2)$
39. $V(0, 0, R) = (4.71 \times 10^{22})G/R$
41. (a) $32/3$
(b) 64π
43. $\sqrt{\pi/5}$
45. (a) It is equal to the average of the values of the function at the vertices.
(b) Same as in (a).
47. Using d for density, the potential is given by $2\pi dG[(\rho_2)^2 - R^2/3 - 2(\rho_1)^3/3R]$.
49. Show that the double integral of $f_{xy} - f_{yx}$ over every rectangle is zero.
51. $u_x^2 + u_y^2 = 1$

Chapter 18 Answers

18.1 Line Integrals

1. 5 **3.** 0 **5.** $3/2$
7. -2π **9.** π **11.** 2π
13. $2\pi^2$ **15.** -1 **17.** 0
19. $2 + e$ **21.** 1
23. $2/3$ **25.** $-1/2$
27. $(\cos 3)/3 + 5/12$ **29.** 0
31. $(5\sqrt{5} - 1)/12$ **33.** $52\sqrt{14}$
35. Use the chain rule and make a change of variables in the integral.

18.2 Path Independence

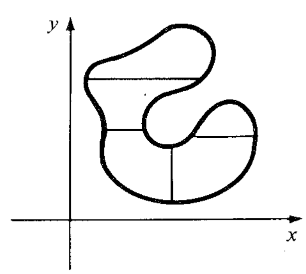
1. 3 **3.** 8 **5.** $1/2$
7. π **9.** 0 **11.** 256
13. $JMm(r_2^{-3} - r_1^{-3})/3$ **17.** Yes; $x^2y + \sin y + C$
15. No **19.** Yes; $x^2 + x^2\cos y + C$
21. Calculate $f_x = -y/(x^2 + y^2)$ and $f_y = x/(x^2 + y^2)$.
23. $2\pi^2$
25. 0
27. (a) The integrals along the four sides are 0, 1, -1, and 0.
(b) $f(x, y, z) = z^3x + x^2y + C$.
29. $(1/2)^3 - (\sqrt{2}/2)^3 + \sin(3\sqrt{2}\,\pi/4)$
31. The field is not conservative.
33. $x\exp(yz) + C$
35. If $\nabla f = \nabla g$ then $\nabla(f - g) = 0$. Show that $f - g$ is constant, using the second box on p. 896.
37. Pick a point P and draw an arc from P to each region. If the arc crosses the circles an even number of times, color the region red, otherwise color the region blue.

18.3 Exact Differentials

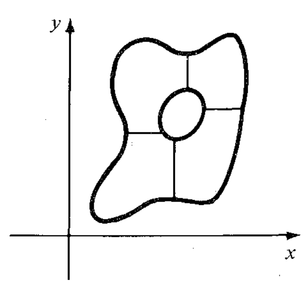
1. No **3.** No
5. Not exact **7.** Exact
9. f is the integral of $P\,dx + Q\,dy$ along the line segment from $(0, 0)$ to $(0, y)$ followed by the segment from $(0, y)$ to (x, y).
11. The two line integrals are equal since $P\,dx + Q\,dy$ is exact.
13. $xe^y + ye^x = 2$
15. $x^3 + x^2y + y^3/3 = 17/3$.
17. Not exact
19. $2x^3y + x^2y^2 = C$
21. (a) -1
(b) $y = x$
23. $y = \pm x$
25. $\mu = x$
27. $x^2/2 + x/y = C$
29. $\ln \mu = \int [(N_x - M_y)/M]\,dy$
31. If f is an antiderivative of $P\,dx + Q\,dy$, then $f_x = \frac{\partial}{\partial x}\int P\,dx$; now integrate.

18.4 Green's Theorem

1. Each side gives $1/12$.
3. Each side gives $-\pi$.
5.



7.



9. $-4\int_{-b}^{b}\int_{(a/b)\sqrt{b^2-y^2}}^{(a/b)\sqrt{b^2-y^2}} xy\,dx\,dy = 0$
11. $67/6$
13. -20
15. 0
17. (a) Recall that a zero dot product implies orthogonality.
(b) Use Green's Theorem.
19. In applying the hint, note that the terms involving $\nabla u \cdot \nabla v$ cancel.
21. 9
23. Simplify $A = \frac{1}{2}\int_C x\,dy - y\,dx$ to $\frac{3a^2}{8}\int_0^{2\pi}\sin^2 2\theta\,d\theta$; now use the double angle formula to integrate.
25. $5/12$
27. A horizontal line segment divides the region into three regions to which Green's theorem applies; see Example 2.
29. 0
31. $\partial\hat{u}/\partial y = Q$ by the fundamental theorem of calculus; similarly for $\partial\hat{u}/\partial x = P$.
33. The device is run around the perimeter of the region and the mechanism evaluates the integral $\frac{1}{2}\int_{\partial D}(x\,dy - y\,dx)$.

18.5 Circulation and Stokes' Theorem

1. -2
3. $4xy/(x^2+y^2)^2$
5. 8
7. 0
9. $z^3\mathbf{i} + e^z\mathbf{j} + (y\sin xy)\mathbf{k}$.
11. $(2/(x^2+y^2+z^2)^2)(x^3\mathbf{i} + (x^2y - yz^2)\mathbf{j} - z^3\mathbf{k})$
13. Let $\mathbf{\Phi} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$, write out curl $(f\mathbf{\Phi})$, and use the product rule for derivatives.
15. $\nabla\times\nabla f = 0$ is a vector identity; the integral of an exact differential about a closed loop is zero.
17. Compute that $\nabla\times\mathbf{\Phi} = 0$ and use Stokes' theorem.
19. $-1/2$
21. Use Stokes' theorem and $\mathbf{\Phi}(\boldsymbol{\sigma}(s))\cdot\boldsymbol{\sigma}'(s) = 0$.
23. $-17\pi\sqrt{2}$
25. $\int_C \mathbf{H}(\mathbf{r})\cdot d\mathbf{r} = \int\int_S(\nabla\times\mathbf{H})\cdot\mathbf{n}\,dA = \int\int_S \mathbf{J}\cdot\mathbf{n}\,dA$
27. The component of the curl of the velocity of the fluid along a vector $\mathbf{n}$ is the circulation around $\mathbf{n}$ per unit area (p. 921); this is maximized when the curl and $\mathbf{n}$ are aligned.
29. Partition the surface into the upper and lower hemispherical pieces; apply Stokes' theorem to each piece and add.

18.6 Flux and the Divergence Theorem

1. $3x^2 - x^2\cos(xy)$
3. $y\cos(xy) - x^2\sin(x^2y)$
5. -4
7. 0
9. (a) A, C
(b) B, D
11. $y\exp(xy) - x\exp(xy) + y\exp(yz)$
13. 3
15. $12\pi/5$
17. 1
19. Use Gauss' theorem and $\mathbf{V}\cdot\mathbf{n} = 0$.
21. Apply the divergence theorem to $f\mathbf{\Phi}$ using $\nabla\cdot(f\mathbf{\Phi}) = \nabla f\cdot\mathbf{\Phi} + f\nabla\cdot\mathbf{\Phi}$.
23. (i) Use the vector identity $\nabla\cdot(\nabla\times\mathbf{H}) = 0$.
(ii) Since charge is conserved, the rate at which charge is entering equals the rate at which charge is leaving; the total flux is therefore zero. By Gauss' theorem, the integral of $\nabla\cdot\mathbf{J}$ over any region is zero, so $\nabla\cdot\mathbf{J} = 0$.
25. (a) Calculate div $\nabla\phi$ on a region excluding a small ball near $\mathbf{q}$.
(b) Use (a).

Review Exercises for Chapter 18

1. $10/3 - 2\cos 2 + 2\sin 2 - 2\sin 1$
3. $2e$
5. -8
7. $1 - e$
9. $-\cos 5 + \cos 3 + (\ln 4)^2/4 + (44\ln 2)/3 + 83/18$
11. (a) $\sin(\ln(5/4)) - \sin(\ln(3/4)) - (\ln(5/4))^2 + (\ln(3/4))^2$
(b) 0
(c) 0
13. $43/54$
15. -1
17. $(e-1)/3$
19. Not conservative
21. Not conservative
23. Yes; $3z^3y + x^2y + C$
25. Exact; $xe^y\sin x + C$
27. Not exact
29. $y\sin x + x^2e^y + 2y = \pi^2/4$
31. $-x + x^2y + y = 1$
33. Not exact
35. Not exact
37. (a) $\int\int_D(1)\,dx\,dy$, the area of D
(b) $\int\int_D(-1)\,dy\,dx$, the negative of the area of D
(c) $\int\int_D(0)\,dy\,dx = 0$
39. curl $\mathbf{\Phi} = [y/(x+z)^2]\mathbf{i} - [y/(x+z)^2]\mathbf{k}$; div $\mathbf{\Phi} = 1/(x+z)$
41. curl $\mathbf{\Phi} = -4x\mathbf{i} - 2y\mathbf{j} + 2z\mathbf{k}$; div $\mathbf{\Phi} = 0$.

43. (a) $\nabla \times \mathbf{F} = 2x^3yz\mathbf{i} - 3x^2y^2z\mathbf{j} + 2\mathbf{k}$, $\nabla \cdot \mathbf{F} = x^3y^2$
(b) 2π
(c) $1/12$

45. $\int\int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dA$, where C is the boundary of S.

47. 0

49. $-2\pi/3$

51. Use Gauss' theorem.

53. Use Gauss' theorem over a small region; divide by the volume of the region and use the mean value theorem for integrals.

55. -8π

57. (a) Let $u' = \int_0^x a(t,0,0)\,dt + \int_0^z c(x,0,t)\,dt + \int_0^y b(x,t,z)\,dt$, so $\partial u'/\partial y = Q$. Permute x, y, z to give u'' with $\partial u''/\partial z = R$, and u''' with $\partial u'''/\partial x = P$. Use Stokes' theorem to show that $u' = u'' = u'''$.
(b) $x^2yz - \cos x + C$

59. $\int\int_{\partial W} f(\nabla f) \cdot \mathbf{n}\, dA = \int\int\int_W (f\nabla^2 f + \nabla f \cdot \nabla f)\,dx\,dy\,dz$ gives $0 = \int\int\int_W (\nabla f \cdot \nabla f)\,dx\,dy\,dz$
$= \int\int\int_W \|\nabla f\|^2\,dx\,dy\,dz$, so $\nabla f = 0$ and thus f is constant.

61. $\int\int_S (\nabla \times \Phi) \cdot \mathbf{n}\, dA = 0$ if S is the union of the two surfaces.